# Final Exam Topics

The final exam will be comprehensive, but will focus more on material from after the second midterm (Sections 4.3, 4.4, 5.1–5.3, 5.5, 6.1, 7.1, and 7.3). Review past homework, quizzes, and exams for more examples and applications of these concepts.

## Chapter 1: Precalculus Skills

Section 1.1:

- Functions via tables, graphs, and formulas
- Domain and range of a function

Section 1.2:

- Linear functions, slope, difference quotient
- Point-slope formula for line through $(x_1, y_1)$ with slope $m$: $y - y_1 = m(x - x_1)$
- Finding equation of line between two points $(x_1, y_1)$ and $(x_2, y_2)$

Section 1.3:

- Average rate of change of $f$ from $a$ to $b$: $\dfrac{\Delta f}{\Delta x} = \dfrac{f(b) - f(a)}{b - a}$
- Graphical interpretation: slope of secant line of $f(x)$ from $a$ to $b$
- Average velocity: $\Delta x / \Delta t$
- Relative change: $\Delta P / P_0$

Section 1.5:

- Exponential functions: constant relative rate of change
- Exponental properties: $a^x a^y = a^{x+y}$, $(a^x)^y = a^{xy}$, $a^0 = 1$
- Formulas: $P(t) = P_0 a^t$, $a$ the growth factor; $P_0(1 + r)^t$, $r$ the growth rate; $P_0 e^{kt}$, $k$ the continuous growth rate ($P_0$ is the initial amount)
- growth: $a > 1$, $r > 0$, $k > 0$; decay: $0 < a < 1$, $-1 < r < 0$, $k < 0$

Section 1.6:

- Logarithm as inverse to exponential: $\log_a y = x$ means $y = a^x$
- Natural logarithm $\ln = \log_e$: $\ln y = x$ means $y = e^x$
- Log properties: $\log_a x + \log_a y = \log_a(xy)$, $\log_a x^p = p \log_a x$, $\log_a a = 1$, $\log_a 1 = 0$

Section 1.7:

- Exponential growth and decay; doubling time (growth) and half-life (decay)

Section 1.8:

- Function composition: $(f \circ g)(x) = f(g(x))$
- Translate $f(x)$: vertical, $f(x) + c$ (up $c$); horizontal, $f(x - c)$ (to right $c$)
- Scaling: vertical stretch by $c$, $cf(x)$; horizontal compression by $c$, $f(cx)$
- Reflection: across $x$-axis, $-f(x)$; across $y$-axis, $f(-x)$

Section 1.9:

- $y$ (directly) proportional to $x$: $y = kx$, $k$ constant
- $y$ inversely proportional to $x$: $y = \dfrac{k}{x}$, or $xy = k$
- Power functions: $y = kx^p$

## Chapter 2: Derivatives

Section 2.1:

- Instantaneous rate of change of $f$ at $a$: $f'(a)$ (number)
- Understand how we get $f'(a)$ from average rates of change over shorter and shorter intervals
- Example: instantaneous velocity from average velocities
- Tangent line to $f(x)$ at $x = a$: $y = f'(a)(x - a) + f(a)$, from point-slope formula
- Understand how the tangent line is the limit of secant lines (picture of lines)
- Estimate $f'(a)$ from graphical or tabular data

Section 2.2:

- $f'(x)$ is itself a function: tells slope of $f(x)$ at different places on graph
- $f'(x) > 0$ means $f(x)$ increasing; $f'(x) < 0$ means $f(x)$ decreasing
- Relate the graph of $f(x)$ to the graph of $f'(x)$

Section 2.3:

- Leibniz notation: $f'(x) = \frac{df}{dx}$, $y'(t) = \frac{dy}{dt}$
- Units and interpretation of the derivative: units of $f'(x)$ = units of $f(x)$/units of $x$
- Tangent line to $f(x)$ at $x = a$ is best linear approximation to $f(x)$ at $a$

Section 2.4:

- The second derivative $f''(x)$: derivative of function $f'(x)$
- $f''(x) > 0$, $y = f(x)$ concave up; $f''(x) < 0$, $y = f(x)$ concave down

## Chapter 3: Derivative Rules

Section 3.1:

- Power Rule: derivative of $x^n$ is $nx^{n-1}$
- Find derivatives of $\dfrac{1}{x^n}$, $\sqrt[n]{x^p}$ using power rule
- Constant Multiple Rule: derivative of $cf(x)$ is $cf'(x)$
- Sum Rule: derivative of $f(x) + g(x)$ is $f'(x) + g'(x)$

Section 3.2:

- Exponential Rule: derivative of $e^x$ is $e^x$
- More exponentials: derivative of $e^{kx}$ is $ke^{kx}$, derivative of $a^x$ is $(\ln a)a^x$
- Logarithm Rule: derivative of $\ln x$ is $\frac{1}{x}$

Section 3.3:

- Chain Rule: derivative of $f(g(x))$ is $f'(g(x))g'(x)$
- Practice decomposing function $H(x)$ as $f(z)$, $z = g(x)$

Section 3.4:

- Product Rule: derivative of $f(x)g(x)$ is $f'(x)g(x) + f(x)g'(x)$
- Quotient Rule: derivative of $\frac{f(x)}{g(x)}$ is $\frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
- Recognize which rule or combination of rules to use for a given function

## Chapter 4: Applications of Derivatives

Section 4.1:

- Definitions of local minima and maxima
- Critical points: $p$ where $f'(p) = 0$ or where $f'(p)$ is undefined
- First Derivative Test: local max/min from change in sign of $f'(x)$ at critical point $p$
- Second Derivative Test: $f'(p) = 0$, $f''(p) < 0$ mean local max, $f''(p) > 0$, local min

Section 4.2:

- Inflection point: change in sign of concavity at $p$
- Check where $f''(p) = 0$ or does not exist, but not a guarantee

Section 4.3:

- Definitions of global maxima and minima
- Finding global extrema of $f(x)$ on a closed interval $[a, b]$: check critical points of $f$ and endpoints $a$ and $b$
- Finding global extrema of $f(x)$ on an open or infinite interval: check critical points of $f$ and behavior of $f$ as it approaches each end of the interval

Section 4.4:

- Profit is revenue minus cost: $P = R - C$
- Revenue is price $\times$ quantity: $R(q) = qp(q)$
- "Marginal" means "derivative of" (e.g., marginal revenue, cost, or profit)
- Find quantity $q$ maximizing revenue: Solve $R'(q) = 0$ for $q$
- Find $q$ maximizing profit: Solve $R'(q) = C'(q)$ (marginal revenue $=$ marginal cost)

## Chapters 5 and 6: Definite Integrals

Section 5.1:

- Distance traveled is time $\times$ velocity
- Graphical over- and underestimates of distance traveled or total change
- Refining estimates by taking smaller time steps

Section 5.2:

- Left and right Riemann sums of $f$ from $a$ to $b$
- Notation and formulas: $n$ (number of subdivisions), $\Delta t$ or $\Delta x = \dfrac{b-a}{n}$ (length of each subdivision), $t_i = a + i\Delta t$ (endpoints of subdivisions, $i$ ranging from 0 to $n$)
- Definite integral $\displaystyle\int_a^b f(x)\,dx$: value is limit of Riemann sums as $n \to \infty$
- Definite integral as area under curve $y = f(x)$ from $a$ to $b$
- Approximate the value of a definite integral with Riemann sums

Section 5.3:

- Integrals as area: area above $x$-axis is positive; area below the $x$-axis, negative
- Relate total area of a region to definite integrals
- Area enclosed by $y = f(x)$ and $x$-axis: find limits with $f(x) = 0$
- Area enclosed by $f(x)$ and $g(x)$: $\displaystyle\int_a^b f(x) - g(x)\,dx$, find limits with $f(x) = g(x)$;

Section 5.5:

- Fundamental Theorem of Calculus: $\displaystyle\int_a^b f'(x)\,dx = f(b) - f(a)$ (total change in $f$)

Section 6.1:

- Average value of $f$ from $a$ to $b$: $\displaystyle\frac{1}{b-a}\int_a^b f(x)\,dx$
- Average rate of change from average value of instantaneous rate of change

## Chapter 7: Antiderivatives

Section 7.1:

- Definition of specific, general ("$+C$") antiderivative of $f(x)$; indefinite integral
- Antiderivative of $x^n$ is $\dfrac{1}{n+1}x^{n+1}$, $n \neq -1$
- Antiderivative of $e^{kx}$ is $\dfrac{1}{k}e^{kx}$; Antiderivative of $\dfrac{1}{x}$ is $\ln x$

Section 7.3:

- Evaluate definite integrals with antiderivatives and the Fundamental Theorem