Homework #3 Solutions

Problems

- Section 1.6: 10, 20, 24, 28, 36
- Section 1.7: 2, 6, 16, 20
- Section 1.8: 2, 8, 14, 20, 22, 28

1.6.10. Solve for t using natural logarithms if $10 = 6e^{0.5t}$.

Solution: We isolate t:

$$\begin{aligned} \frac{10}{6} &= e^{0.5t} \\ \ln\left(\frac{10}{6}\right) &= \ln(e^{0.5t}) = 0.5t \\ t &= \frac{\ln\left(\frac{10}{6}\right)}{0.5} = 2\ln\left(\frac{5}{3}\right) \end{aligned}$$

Evaluating this numerically, $t \approx 1.022$. ∎

1.6.20. The function $P = 3.2e^{0.03t}$ represent exponential growth or decay. What is the initial quantity? What is the growth rate? State if the growth rate is continuous.

Solution: The initial quantity is 3.2, and the (continuous) growth rate is 0.03. ∎

1.6.24. Write the function $P = 2e^{-0.5t}$ in the form $P = P_0 a^t$. Does the function represent exponential growth or decay?

Solution: The continuous growth rate k is -0.5, so the growth factor a is $e^{-0.5} \approx 0.607$. Since $k < 0$, this function represents decay. ∎

1.6.28. Put the function $P = 10(1.7)^t$ in the form $P = P_0 e^{kt}$.

Solution: In this case, the growth factor a is 1.7, so $k = \ln(1.7) \approx 0.531$. ∎

1.6.36. The gross world product is $W = 32.4(1.036)^t$, where W is in trillions of dollars and t is years since 2001. Find a formula for gross world product using a continuous growth rate.

Solution: The growth factor is $a = 1.036$, so the continuous growth rate is $k = \ln(1.036) \approx 0.0354$. Hence, the new formula is

$$W = 32.4e^{0.0354t}.$$ ■

1.7.2. The half-life of nicotine in the blood is 2 hours. A person absorbs 0.4 mg of nicotine by smoking a cigarette. Fill in the following table with the amount of nicotine in the blood after t hours. Estimate the length of time until the amount of nicotine is reduced to 0.04 mg.

Solution:

t (hours)	0	2	4	6	8	10
Nicotine (mg)	0.4	0.2	0.1	0.05	0.025	0.0125

A formula describing the amount of nicotine at time t is $N(t) = 0.4(2)^{-t/2}$. Then $0.04 = 0.4(2)^{-t/2}$, so $0.1 = 2^{-t/2}$, and $-t/2 = \log_2(0.1)$. Finally, $t = -2\log_2(0.1) \approx 6.64$ hours. ■

1.7.6. Suppose 1000 is invested in an account paying interest at a rate of 5.5% per year. How much is in the account after 8 years if the interest is compounded

(a) Annually?

(b) Continuously?

Solution (a): The growth reate is 0.055, so the amount is $1000(1.055)^8 \approx 1534.69$. ■

Solution (b): The continuous growth rate is 0.055, so the amount is $1000e^{0.055(8)} \approx 1552.71$. ■

1.7.16. The antidepressant fluoxetine (or Prozac) has a half-life of about 3 days. What percentage of a dose remains in the body after one day? After one week?

Solution: We write a function $P(t)$ that gives the proportion of Prozac remaining after time t. Since the half-life is 3 days, $P(t) = 2^{-t/3}$. Then $P(1) = 2^{-1/3} \approx 0.794 = 79.4\%$ is the percentage remaining after 1 day, and $P(7) = 2^{-7/3} \approx 0.198 = 19.8\%$ is the percentage remaining after 7 days. ■

1.7.20. The number of people living with HIV infections increased worldwide approximately exponentially from 2.5 million in 1985 to 37.8 million in 2003.

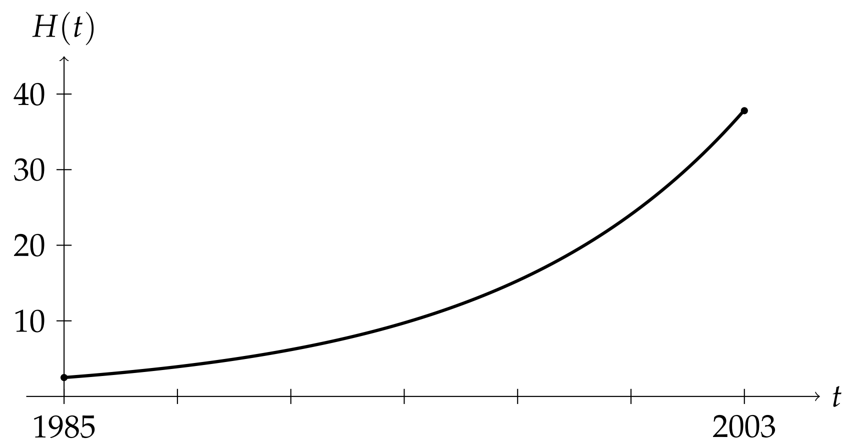
(a) Give a formula for the number of HIV infections, H, (in millions) as a function of years, t, since 1985. Use the form $H = H_0 e^{kt}$. Graph the function.

(b) What was the yearly continuous percent change in the number of HIV infections between 1985 and 2003?

Solution (a): We have that $H_0 = 2.5$. In 2003, $t = 2003 - 1985 = 18$, so we have $37.8 = 2.5e^{18k}$. Then $k = \ln\left(\frac{37.8}{2.5}\right)/18 \approx 0.151$, so the formula is

$$H(t) = 2.5e^{0.151t}.$$ ∎

The graph of this function is as follows:



Solution (b): The continuous percentage change is 15.1% per year. ∎

1.8.2. If $f(x) = x^2 + 1$, find and simplify:

(a) $f(t+1)$

(b) $f(t^2+1)$

(c) $f(2)$

(d) $2f(t)$

(e) $[f(t)]^2 + 1$

Solution (a): $f(t+1) = (t+1)^2 + 1 = t^2 + 2t + 1 + 1 = t^2 + 2t + 2.$ ∎

Solution (b): $f(t^2+1) = (t^2+1)^2 + 1 = t^4 + 2t^2 + 2.$ ∎

Solution (c): $f(2) = 2^2 + 1 = 5.$ ∎

Solution (d): $2f(t) = 2(t^2 + 1) = 2t^2 + 2.$ ■

Solution (e): $[f(t)]^2 + 1 = (t^2 + 1)^2 + 1 = t^4 + 2t^2 + 2.$ ■

1.8.8. Find the following if $f(x) = 2x^2$ and $g(x) = x + 3$:
(a) $f(g(x))$
(b) $g(f(x))$
(c) $f(f(x))$

Solution (a): $f(g(x)) = 2(x + 3)^2 = 2x^2 + 12x + 18.$ ■

Solution (b): $g(f(x)) = 2x^2 + 3.$ ■

Solution (c): $f(f(x)) = 2(2x^2)^2 = 8x^4.$ ■

1.8.14. Use the variable u for the inside function to express each of the following as a composite function:
(a) $y = (5t^2 - 2)^6$
(b) $P = 12e^{-0.6t}$
(c) $C = 12\ln(q^3 + 1)$

Solution (a): Set $u = 5t^2 - 2$, so $y = u^6$. ■

Solution (b): Set $u = -0.6t$, so $P = 12e^u$. ■

Solution (c): Set $u = q^3 + 1$, so $C = 12\ln u$. ■

1.8.20. Estimate $g(f(2))$ from the graphs of f and g.

Solution: First, from the graph of f above $x = 2$, $f(2) \approx 0.4$. Then from the graph of g above $x = 0.4$, $g(0.4) \approx 1.2$. ■

1.8.22. Using Table 1.36, create a table of values for $f(g(x))$ and for $g(f(x))$.

x	-3	-2	-1	0	1	2	3
$f(x)$	0	1	2	3	2	1	0
$g(x)$	3	2	2	0	-2	-2	-3

Solution: We calculate each value: for example, for $f(g(-1))$, we first look up $g(-1) = -2$. Then $f(g(-1)) = f(-2)$, so we look that up to find $f(-2) = 1$. Hence, $f(g(-1)) = 1$.

x	-3	-2	-1	0	1	2	3
$f(g(x))$	0	1	1	3	1	1	0
$g(f(x))$	0	-2	-2	-3	-2	-2	0

■

1.8.28. The Heaviside step function, H, is graphed in Figure 1.79. Graph the following functions:

(a) $2H(x)$

(b) $H(x)+1$

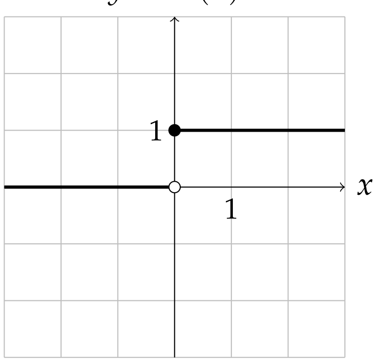
(c) $H(x+1)$

(d) $-H(x)$

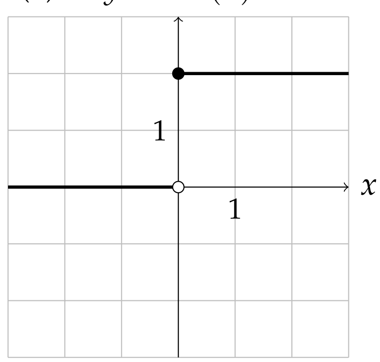
(e) $H(-x)$

Solution: Below are the graphs:

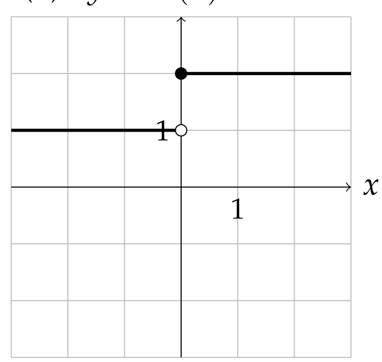
$y = H(x)$



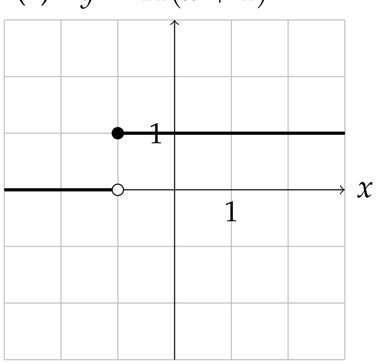
(a) $y = 2H(x)$



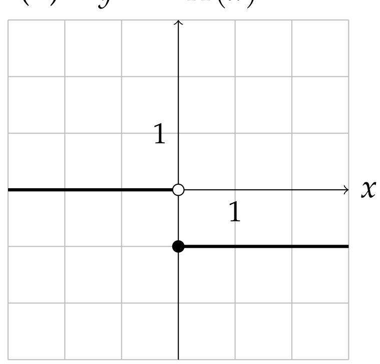
(b) $y = H(x)+1$



(c) $y = H(x+1)$



(d) $y = -H(x)$



(e) $y = H(-x)$