Homework #10 Solutions

Problems

Bolded problems are worth 2 points.

- Section 5.1: 4, 6, **8**, **12**, 14
- Section 5.2: **2**, 6, 10, **18**, 22, **30**
- Notes: On 5.2.22, evaluate the integral using the `fnInt` function (available through [MATH] [9]) on a TI graphing calculator, or through another computation system. On 5.2.30, the sums should cover the interval from $t = 15$ to $t = 23$.

5.1.4. A car starts moving at time $t = 0$ and goes faster and faster. Its velocity is shown in the following table. Estimate how far the car travels during the 12 seconds.

t (sec)	0	3	6	9	12
v (ft/sec)	0	10	25	45	75

Solution: Since the car is accelerating, its velocity function is increasing, so we can get an underestimate for the distance traveled by taking a left-hand sum over 3-second intervals:

$$L = 0 \cdot 3 + 10 \cdot 3 + 25 \cdot 3 + 45 \cdot 3 = 240 \text{ ft.}$$

Similarly, we can get an overestimate with a right-hand sum:

$$L = 10 \cdot 3 + 25 \cdot 3 + 45 \cdot 3 + 75 \cdot 3 = 465 \text{ ft.}$$

A better estimate is usually obtained from averaging the left- and right-hand estimates, which in this case gives $\dfrac{240 + 465}{2} = 352.5$ ft. ■

5.1.6. Figure 5.8 shows the velocity, v, of an object (in meters/sec). Estimate the total distance the object traveled between $t = 0$ and $t = 6$.

Solution: We estimate values for $v(t)$ at $t = 0, 1, \ldots, 6$:

t	0	1	2	3	4	5	6
$v(t)$	0	14	21	26	30	34	38

Then the left-hand and right-hand sums for these values are

$$L = 0 + 14 + 21 + 26 + 30 + 34 = 125, \qquad R = 14 + 21 + 26 + 30 + 34 + 38 = 163,$$

and their average is $\frac{1}{2}(125 + 163) = 144$ m. Estimates may differ if different values of $v(t)$ are determined. ■

5.1.8. The following table gives world oil consumption, in billions of barrels per year. Estimate total oil consumption during this 25-year period.

Year	1980	1985	1990	1995	2000	2005
Oil (bn bbl/yr)	22.3	21.3	23.9	24.9	27.0	29.3

Solution: We again average the left-hand and right-hand sum estimates for these data:

$$L = 22.3 \cdot 5 + 21.3 \cdot 5 + 23.9 \cdot 5 + 24.9 \cdot 5 + 27.0 \cdot 5 = 597.0,$$
$$R = 21.3 \cdot 5 + 23.9 \cdot 5 + 24.9 \cdot 5 + 27.0 \cdot 5 + 29.3 \cdot 5 = 632.0.$$

Then their average is $\frac{1}{2}(597.0 + 632.0) = 614.5$ billion barrels. ■

5.1.12. Roger runs a marathon. His friend Jeff rides behind him on a bicycle and clocks his speed every 15 minutes. Roger starts out strong, but after an hour an a half he is so exhausted that he has to stop. Jeff's data follow:

Time (min)	0	15	30	45	60	75	90
Speed (mph)	12	11	10	10	8	7	0

(a) Assuming that Roger's speed is never increasing, give upper and lower estimates for the distance Roger ran during the first half-hour.

(b) Give upper and lower estimates for the distance Roger ran during the entire hour and a half.

Solution (a): Since Roger is decelerating, his velocity is decreasing, so a left-hand sum will give us an overestimate (and a right-hand one, an underestimate). To make the units correct, we convert the time intervals from 15 minutes to $\frac{1}{4}$ of an hour when we compute the sum. For the first half-hour, we use only two intervals:

$$L = 12 \cdot \frac{1}{4} + 11 \cdot \frac{1}{4} = \frac{23}{4} = 5.75, \qquad R = 11 \cdot \frac{1}{4} + 10 \cdot \frac{1}{4} = \frac{21}{4} = 5.25.$$

Therefore, Roger traveled at least 5.25 miles and at most 5.75. ■

Solution (b): We take left- and right-hand sums for the entire 90-minute interval:

$$L = \frac{1}{4}(12 + 11 + 10 + 10 + 8 + 7) = 14.5, \quad R = \frac{1}{4}(11 + 10 + 10 + 8 + 7 + 0) = 11.5.$$

Therefore, Roger ran somewhere between 11.5 and 14.5 miles. ■

5.1.14. Figure 5.10 shows the rate of change of a fish population. Estimate the total change in the population during this 12-month period.

Solution: We use the graph to estimate the rate of change every 2 months:

t (months)	0	2	4	6	8	10	12
$r(t)$ (fish/month)	10	17	21	22	21	17	10

Then averaging the left- and right-hand sums, we have

$$L = (10 + 17 + 21 + 22 + 21 + 17) \cdot 2 = 216,$$
$$R = (17 + 21 + 22 + 21 + 17 + 10) \cdot 2 = 216,$$

so the total change in population is probably about 216 fish. ∎

5.2.2. Estimate $\int_0^{12} \frac{1}{x+1}\,dx$ using a left-hand sum with $n = 3$.

Solution: With $a = 0$, $b = 12$, and $n = 3$, $\Delta x = \frac{12-0}{3} = 4$. Then the left-hand sum uses the x-values 0, 4, and 8, so it is

$$\frac{1}{1+0} \cdot 4 + \frac{1}{1+4} \cdot 4 + \frac{1}{1+8} \cdot 4 = 4 + \frac{4}{5} + \frac{4}{9} = \frac{236}{45} \approx 5.2444.$$ ∎

5.2.6. Use the table to estimate $\int_0^{40} f(x)\,dx$. What values of n and Δx did you use?.

x	0	10	20	30	40
$f(x)$	350	410	435	450	460

Solution: Since the x-values in the table are spaced every 10 units, we use $\Delta X = 10$. Since there are 5 values total, we use $n = 5 - 1 = 4$ (since each sum ignores either a or b). Then the left- and right-hand sums gives estimates:

$$L = (350 + 410 + 435 + 450) \cdot 10 = 16{,}450, \quad R = (410 + 435 + 450 + 460) \cdot 10 = 17{,}550.$$

Averaging them, we have the estimate $\frac{1}{2}(16{,}450 + 17{,}550) = 17{,}000$. ∎

5.2.10. Use the graph provided to estimate $\int_0^3 f(x)\,dx$.

Solution: We estimate $f(x)$ at multiples of $\frac{1}{2}$ between 0 and 3:

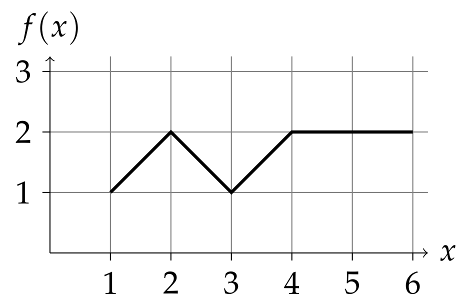
x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$f(x)$	4.0	5.2	6.6	7.2	6.5	5.0	3.2

These estimates are probably accurate to ± 0.1. Then the left- and right-hand sums are

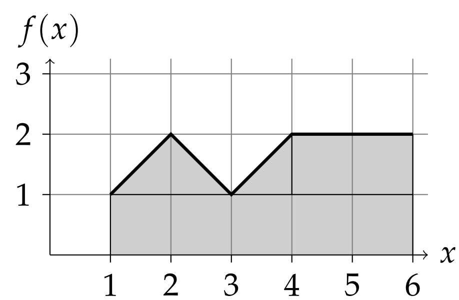
$$L = (4.0 + 5.2 + 6.6 + 7.2 + 6.5 + 5.0) \cdot \frac{1}{2} = 17.25,$$
$$R = (5.2 + 6.6 + 7.2 + 6.5 + 5.0 + 3.2) \cdot \frac{1}{2} = 16.85.$$

The average is $\frac{1}{2}(17.25 + 16.85) = 17.05$. (If the estimates of the $f(x)$ values differ, or if the x-values where the $f(x)$ estimates are taken change, this estimate will vary as well.) ∎

5.2.18. Using the figure below, find the value of $\int_1^6 f(x)\,dx$.



Solution: We estimate the area under the figure from $x = 1$ to $x = 6$ geometrically. Shading the region under the curve and cutting it into rectangles and squares, we have



The two rectangles have area 5 and 2, the triangle on the left has area $\frac{1}{2}(1)(2) = 1$, and the one on the right has area $\frac{1}{2}(1)(1) = \frac{1}{2}$. Hence, the total area is $5 + 2 + 1 + \frac{1}{2} = 8.5$, so this is the value of the integral. ∎

5.2.22. Use a calculator to evaluate $\displaystyle\int_1^4 \frac{1}{\sqrt{1+x^2}}\,dx$.

Solution: Using the `fnInt` function on the TI, we find that

$$\int_1^4 \frac{1}{\sqrt{1+x^2}}\,dx \approx 1.21334.$$

∎

5.2.30. Use the data in the table below from $t = 15$ to $t = 23$ and the notation for Riemann sums.

t	15	17	19	21	23
$f(t)$	10	13	18	20	30

(a) If $n = 4$, what is Δt? What are t_0, t_1, t_2, t_3, t_4? What are $f(t_0), f(t_1), f(t_2), f(t_3), f(t_4)$?

(b) Find the left and right sums using $n = 4$.

(c) If $n = 2$, what is Δt? What are t_0, t_1, t_2? What are $f(t_0), f(t_1), f(t_2)$?

(d) Find the left and right sums using $n = 2$.

Solution (a): For $n = 4$, $\Delta t = \dfrac{23-15}{4} = 2$. Then $t_0 = 15$, $t_1 = 17$, $t_2 = 19$, $t_3 = 21$, and $t_4 = 23$, so $f(t_0) = 10$, $f(t_1) = 13$, $f(t_2) = 18$, $f(t_3) = 20$, and $f(t_4) = 30$. ■

Solution (b): The left and right sums with $n = 4$ are

$$\sum_{i=0}^{3} f(t_i)\Delta t = 10 \cdot 2 + 13 \cdot 2 + 18 \cdot 2 + 20 \cdot 2 = 122,$$
$$\sum_{i=1}^{4} f(t_i)\Delta t = 13 \cdot 2 + 18 \cdot 2 + 20 \cdot 2 + 30 \cdot 2 = 162.$$ ■

Solution (c): For $n = 2$, $\Delta t = \dfrac{23-15}{2} = 4$. Then $t_0 = 15$, $t_1 = 19$, and $t_2 = 23$, so $f(t_0) = 10$, $f(t_1) = 18$, and $f(t_2) = 30$. ■

Solution (d): The left and right sums with $n = 2$ are

$$\sum_{i=0}^{1} f(t_i)\Delta t = 10 \cdot 4 + 18 \cdot 4 = 112, \qquad \sum_{i=1}^{2} f(t_i)\Delta t = 18 \cdot 4 + 30 \cdot 4 = 192.$$ ■