Homework #4 Solutions

Problems

- Section 1.9: 4, 10, 14, 18, 24, 30
- Section 2.1: 6, 10, 16, 18
- Section 2.2: 4, 10, 22, 24, 28

1.9.4. Determine if $y = \dfrac{3}{8x}$ is a power function, and if it is, write it in the form kx^p and give the values of k and p.

Solution: We have that $y = \dfrac{3}{8x} = \dfrac{3}{8} \cdot \dfrac{1}{x} = \dfrac{3}{8}x^{-1}$, so this is a power function, with $k = \dfrac{3}{8}$ and $p = -1$. ∎

1.9.10. Determine if $y = (5x)^3$ is a power function, and if it is, write it in the form kx^p and give the values of k and p.

Solution: We have that $y = (5x)^3 = 5^3x^3 = 125x^3$, so this is a power function, with $k = 125$ and $p = 3$. ∎

1.9.14. Write a formula for the energy, E, expended by a swimming dolphin, which is proportional to the cube of the speed, v, of the dolphin.

Solution: The energy E is proportional to v^3, so $E = kv^3$ for some constant k. ∎

1.9.18. The surface area of a mammal, S, satisfies the equation $S = kM^{2/3}$, where M is the body mass, and the constant of proportionality k depends on the body shape of the mammal. A human of body mass 70 kilograms has surface area 18,600 cm^2. Find the constant of proportionality for humans. Find the surface area of a human with body mass 60 kilograms.

Solution: From the data given, $18{,}600 = k(70)^{2/3}$, so

$$k = \frac{18{,}600}{(70)^{2/3}} \approx 1095.1.$$

Then the surface area of a 60-kilogram human is $S = k(60)^{2/3} \approx (1095.1)(60)^{2/3} \approx 16{,}784$ cm^2. ∎

1.9.24. The specific heat, s, of an element is the number of calories of heat required to raise the temperature of one gram of the element by one degree Celsius. Use the following table to decide if s is proportional or inversely proportional to the atomic weight, w, of the element. If so, find the constant of proportionality.

Element	Li	Mg	Al	Fe	Ag	Pb	Hg
w	6.9	24.3	27.0	55.8	107.9	207.2	200.6
s	0.92	0.25	0.21	0.11	0.056	0.031	0.033

Solution: If s is proportional to w, then $s = kw$, so $s/w = k$ is constant. If instead s is inversely proportional to w, then $s = k/w$, so $sw = k$ is constant. Hence, we tabulate both sw and s/w for these values:

Element	Li	Mg	Al	Fe	Ag	Pb	Hg
s/w	0.133	0.103	0.0078	0.0020	0.00052	0.00015	0.00016
sw	6.35	6.08	5.67	6.14	6.04	6.42	6.62

Since sw is approximately constant while s/w decreases significantly, s is inversely proportional to w. The proportionality parameter k is not quite constant, but averaging the values in the table above gives an estimate of 6.19. ■

1.9.30. A sporting goods wholesaler finds that when the price of a product is \$25, the company sells 500 units per week. When the price is \$30, the number sold per week decreases to 460 units.

(a) Find the demand, q, as a function of price, p, assuming that the demand curve is linear.

(b) Use your answer to part (a) to write revenue as a function of price.

(c) Graph the revenue function in part (b). Find the price that maximizes revenue. What is the revenue at this price?

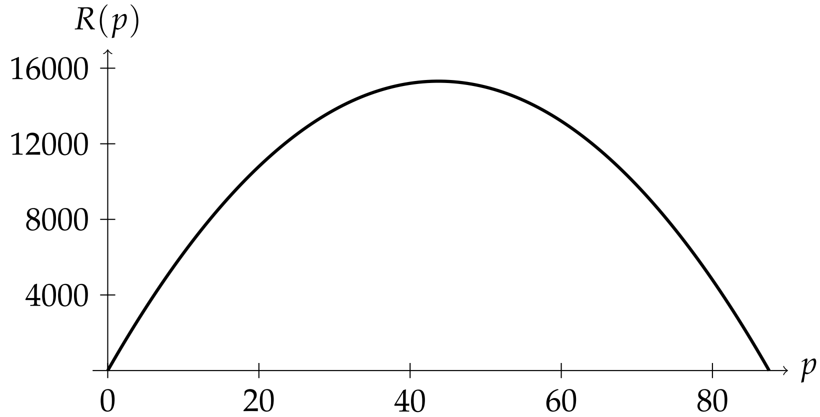
Solution (a): The slope of the linear demand function $q(p)$ is given by

$$m = \frac{\Delta q}{\Delta p} = \frac{460 - 500}{30 - 25} = \frac{-40}{5} = -8.$$

Therefore, the function is $q(p) = -8(p - 25) + 500 = -8p + 200 + 500 = 700 - 8p$. ■

Solution (b): The revenue as a function of price, $R(p)$, is p times $q(p)$, so from part (a) it is $R(p) = p(700 - 8p) = 700p - 8p^2$. ■

Solution (c): The revenue function $R(p)$ is quadratic in the variable p with a negative coefficient on the p^2 term, so it has the shape of an inverted parabola. Since $R(p)$ factors as $p(700 - 8p)$, its roots are $p = 0$ and $p = 700/8 = 87.5$, so we expect the graph to be positive between these two values. We use this to graph the function:



The graph reaches its maximum at the vertex of the parabola, which by symmetry is located halfway between the two roots, at $p = \frac{1}{2}(87.5) = 43.75$. Hence, this price maximizes revenue, and the maximum is $R(43.75) = 700(43.75) - 8(43.75)^2 = 15{,}312.50$. ∎

2.1.6. Figure 2.12 shows the cost, $y = f(x)$, of manufacturing x kilograms of a chemical.

(a) Is the average rate of change of the cost greater between $x = 0$ and $x = 3$, or between $x = 3$ and $x = 5$? Explain your answer graphically.

(b) Is the instantaneous rate of change of the cost of producing x kilograms greater at $x = 1$ or $x = 4$? Explain your answer graphically.

(c) What are the units of these rates of change?

Solution (a): The secant line from $x = 0$ and $x = 3$, between the points $(0, 1)$ and $(3, 3.5)$, has slope $\frac{3.5-1}{3-0} = \frac{2.5}{3} \approx 0.833$. The secant line from $x = 3$ and $x = 5$, between the points $(3, 3.5)$ and $(5, 4.1)$, has slope $\frac{4.1-3.5}{5-3} = \frac{0.6}{2} \approx 0.3$. Therefore, the first rate of change is greater. You can also see this from the graph: the first secant line is much steeper than the second. ∎

Solution (b): The tangent line at $x = 1$ is steeper than that at $x = 4$, so the first instantaneous rate of change is greater. ∎

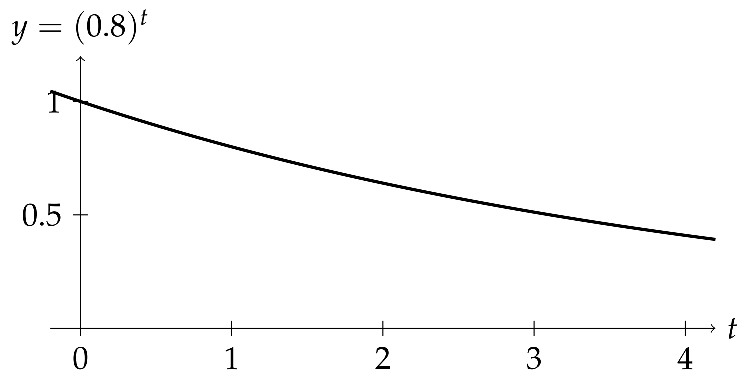
Solution (c): These rates of change are in $\dfrac{\text{units of } f}{\text{units of } x}$, or $\dfrac{\text{dollars}}{\text{kg}}$. ∎

2.1.10.

(a) Let $g(t) = (0.8)^t$. Use a graph to determine whether $g'(2)$ is positive, negative, or zero.

(a) Use a small interval to estimate $g'(2)$.

Solution (a): We graph $g(t)$ around $t = 2$:



Since the tangent line at $t = 2$ slopes downward, $g'(2)$ will be negative. ■

Solution (b): We choose $\Delta t = 0.1$ to estimate $g'(2)$. Then the t-coordinate of the other point on the secant line will be $2 + 0.1 = 2.1$, so the slope of the secant line is

$$g'(2) \approx \frac{g(2.1) - g(2)}{2.1 - 2} = \frac{(0.8)^{2.1} - (0.8)^2}{0.1} = \frac{0.6259 - 0.64}{0.1} = \frac{-0.0141}{0.1} = -0.141.$$

Hence, we estimate $g'(2)$ to be -0.141. (Answers may vary based on the size of the interval chosen, but they should all be approximately the true value of $g'(2) = (\ln 0.8)(0.8)^2 \approx -0.143$.) ■

2.1.16. Table 2.4 gives $P = f(t)$, the percent of households in the US with cable television t years since 1990.

t (years since 1990)	0	2	4	6	8	10	12
P (% with cable)	59.0	61.5	63.4	66.7	67.4	67.8	68.9

(a) Does $f'(6)$ appear to be positive or negative? What does this tell you about the percent of households with cable television?

(b) Estimate $f'(2)$. Estimate $f'(10)$. Explain what each is telling you, in terms of cable television.

Solution (a): Since f increases both from $t = 4$ to $t = 6$ and from $t = 6$ to $t = 8$, $f'(6)$ is probably positive. This says that the percentage of households with cable is increasing. ■

Solution (b): We estimate $f'(2)$ first using a forward step. The next data point we have after $t = 2$ is $t = 4$, so we use that:

$$f'(2) \approx \frac{f(4) - f(2)}{4 - 2} = \frac{63.4 - 61.5}{2 - 0} = \frac{1.9}{2} = 0.95$$

On the other hand, we get a different estimate if we take a backward step, to $t = 0$:

$$f'(2) \approx \frac{f(2) - f(0)}{2 - 0} = \frac{61.5 - 59.0}{4 - 2} = \frac{2.5}{2} = 1.25.$$

We can then even average these two estimates: $\frac{1}{2}(1.25 + 0.95) = 1.1$. This says the percentage of households with cable in increasing by 1.1% per year in 1992.

Similarly, we compute the same averages around $t = 10$: the forward-step estimate is 0.55, the backward-step estimate is 0.2, and their average is 0.375. This says the percentage of households with cable in increasing by 0.375% per year in 2000. ■

2.1.18. The function in Figure 2.15 has $f(4) = 25$ and $f'(4) = 1.5$. Find the coordinates of the points A, B, and C.

Solution: The equation of the tangent line, which is tangent to the graph at $x = 4$, is

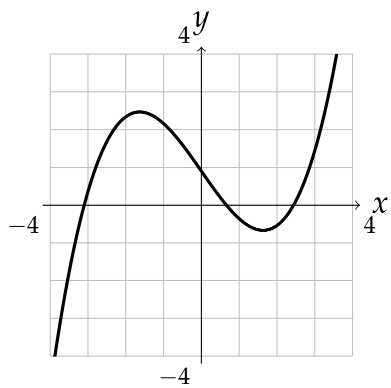
$$y = 1.5(x - 4) + 25.$$

The point A is the point of tangency, so its coordinates are $(4, 25)$.

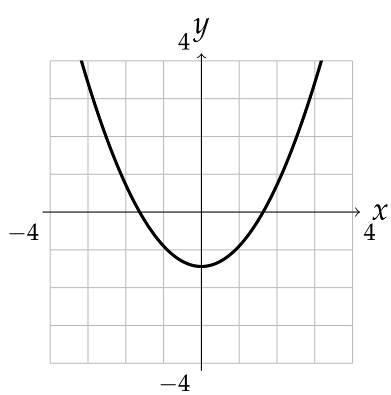
The point B has x-coordinate 4.2, so its y-coordinate is $1.5(4.2 - 4) + 25 = 1.5(0.2) + 25 = 0.3 + 25 = 25.3$.

The point C has x-coordinate 3.9, so its y-coordinate is $1.5(3.9 - 4) + 25 = 1.5(-0.1) + 25 = -0.15 + 25 = 24.85$. ■

2.2.4. Graph the derivative of the given function.



Solution: We observe that the graph is increasing between -4 and approximately -1.6, so it should have a positive there. It decreases between -1.6 and 1.6, so it should have a negative derivative there, and then it increases again after that. At -1.6 and 1.6, it has flat tangent lines, so the derivative should be 0 there.



■

2.2.10. In the graph of f in Figure 2.23, at which of the labeled x-values is:
(a) $f(x)$ greatest?
(b) $f(x)$ least?
(c) $f'(x)$ greatest?
(d) $f'(x)$ least?

Solution (a): Checking the values of f at the x_i, f is greatest at x_3. ∎

Solution (b): f is least at x_4. ∎

Solution (c): f has the steepest positive slope at x_5, so $f'(x)$ is greatest there. ∎

Solution (d): The only x_i at which f is decreasing is x_3, so this is where $f'(x)$ is the least. ∎

2.2.22. Match the graph with its derivative.

Solution: The graph of $f(x)$ is a line of negative slope, so its derivative $f'(x)$ should be constant and negative. The graph (IV) is the best match. ∎

2.2.24. Match the graph with its derivative.

Solution: The graph of $f(x)$ is increasing until approximately $x = -2$, has a horizontal tangent at $x = -2$, and then decreases for x past -2, approaching another horizontal tangent. Therefore, the derivative should be positive until it reaches 0 at $x = -2$, and then it should be negative but again approaching 0 as x increases. Graph (VI) seems to fit the best. ∎

2.2.28.
(a) Let $f(x) = \ln x$. Use small intervals to estimate $f'(1)$, $f'(2)$, $f'(3)$, $f'(4)$, and $f'(5)$.
(b) Use your answers to part (a) to guess a formula for the derivative of $f(x) = \ln x$.

Solution (a): For the sake of simplicity, we use a Δx of 0.01 for each a value. For $f'(1)$, we compute

$$f'(1) \approx \frac{\ln(1.01) - \ln 1}{1.01 - 1} \approx \frac{0.00995 - 0}{0.01} = 0.995.$$

We also do $f'(2)$:

$$f'(2) \approx \frac{\ln(2.01) - \ln 2}{2.01 - 2} \approx \frac{0.69813 - 0.69315}{0.01} = 0.498.$$

Similarly, $f'(3) \approx 0.333$, $f'(4) \approx 0.250$, and $f'(5) \approx 0.200$. ∎

Solution (b): From this pattern, we might guess that $f'(x) = \dfrac{1}{x}$. ∎