Midterm #2 Practice Problems

1. Find general solutions to the following DEs:

(a) $y'' - 6y' + 8y = 0$

(b) $y^{(3)} + 2y'' - 4y' - 8y = 0$

(c) $y'' + 8y' + 20y = 0$

(d) $y^{(4)} = y$

(e) $y^{(6)} + 8y^{(4)} + 16y'' = 0$

2. Find solutions to the following IVPs:

(a) $y'' - 3y' + 2y = 0, y(0) = 1, y'(0) = 0$

(b) $9y^{(3)} + 12y'' + 4y' = 0, y(0) = 0, y'(0) = 1, y''(0) = \frac{10}{3}$

3. For each of the DEs below, find a particular solution to it:

(a) $y'' - 6y' + 8y = 4x + 5$

(b) $y^{(4)} + 4y'' = 12x - 16 - 8e^{2x}$

(c) $y'' + 2y' - 3y = -4xe^{-3x}$

4. Consider a mass of 2 kg attached to a spring with spring constant 18 N/m. Find the displacement $x(t)$ with the initial conditions $x(0) = 4$ m, $x'(0) = 9$ m/s, assuming the following damping c is present. If the system exhibits periodic behavior, find its (possibly time-varying) amplitude and period.

(a) $c = 0$

(b) $c = 4$

(c) $c = 12$

(d) $c = 20$

5. Find a differential equation with the general solution

$$y = (c_1 + c_2 x)e^{3x} + c_3 e^{-2x}\cos(\sqrt{2}x) + c_4 e^{-2x}\sin(\sqrt{2}x).$$

6.

(a) Show that the functions $y_1 = e^{-x}$ and $y_2 = \sin x$ are linearly independent.

(b) Show that the functions $y_1 = 1 + \tan^2 x, y_2 = 3 - 2\tan^2 x$, and $y_3 = \sec^2 x$ are not linearly independent.

7. Find the form of a particular solution to the DE $y^{(3)} - 2y'' + 2y' = 6e^x + 3e^x \sin x - x^2$, but do not determine the values of the coefficients.

8. Consider the nonhomogeneous DE $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 8x^{3/2}\sin x$.

(a) Verify that $y_1 = x^{-1/2}\cos x$ and $y_2 = x^{-1/2}\sin x$ are linearly independent solutions to the associated homogeneous DE.

(b) Use variation of parameters to find a general solution to the nonhomogeneous DE. (Hint: Use the identities $\sin x \cos x = \frac{1}{2}\sin 2x$ and $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.)

9. A spring is stretched 6 inches by a mass m that weighs 8 lb. The mass is attached to a dashpot that has a damping force of 2 lb-s/ft, and an external force of $4\cos 2t$ lb acts on it.

(a) Describe the steady state response of the system (that is, the particular solution to the nonhomogeneous equation).

(b) Find the value of the mass m that maximizes the amplitude of this response, with all other parameters remaining constant. What is this maximum amplitude? What does this mass weigh in pounds?