

Euler Method Functions

```
EulerMethodPoints[n_, h_, x0_, y0_, f_] := Module[
  {i = 0, pts = {{x0, y0}}, point},
  For[, i < n, i++,
    point = Last[pts];
    AppendTo[pts, point + {h, h Apply[f, point]}];
  ];
  pts
]
EulerMethodPointsN[n_, h_, x0_, y0_, f_] := Module[
  {i = 0, pts = {{x0, y0}}, point},
  For[, i < n, i++,
    point = Last[pts];
    AppendTo[pts, point + N[{h, h Apply[f, point]}]];
  ];
  pts
]
EulerMethodTable[n_, h_, x0_, y0_, f_] := Module[
  {i = 0, pts = {Prepend[N[{x0, y0, f[x0, y0]}], 0]}, point, newpoint, entry},
  For[, i < n, i++,
    entry = Last[pts];
    point = entry[[2 ;; 3]];
    newpoint = point + {h, h entry[[4]]};
    AppendTo[pts, Prepend[N[Append[newpoint, Apply[f, newpoint]]], i + 1]];
  ];
  TraditionalForm[
    Grid[Prepend[pts, {"n", "xn", "yn", "yn'"}], Frame → All, Alignment → "."]]
]
```

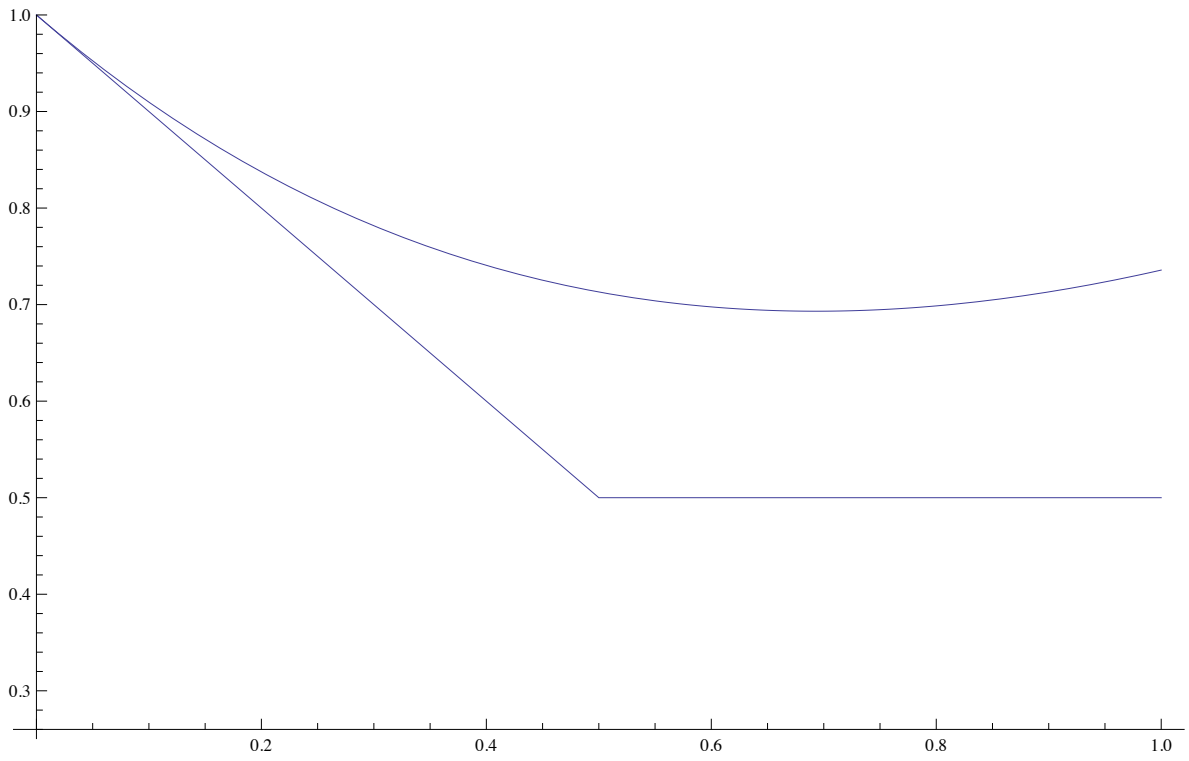
IVP : $y' = x - y$, $y(0) = 1$; estimate $y(1)$

```
f1[x_, y_] = x - y;
solexact1[x_] = x - 1 + 2 e-x;
solexact1[1] // N
0.735759

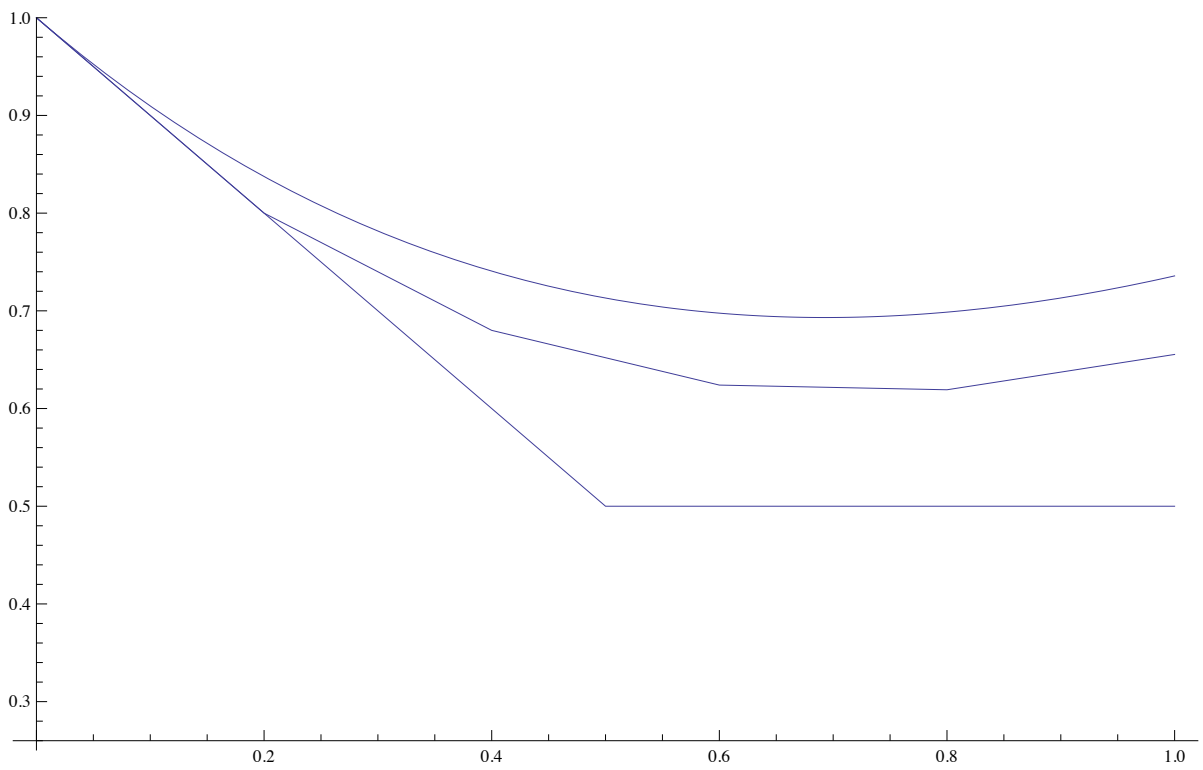
plotexact1 = Plot[solexact1[x], {x, 0, 1}, PlotRange → {1/4, 1}];

plotleuler2 = ListLinePlot[EulerMethodPoints[2, 1/2, 0, 1, f1]];
plotleuler5 = ListLinePlot[EulerMethodPoints[5, 1/5, 0, 1, f1]];
plotleuler10 = ListLinePlot[EulerMethodPoints[10, 1/10, 0, 1, f1]];
plotleuler100 = ListLinePlot[EulerMethodPoints[100, 1/100, 0, 1, f1]];
plotleuler1000 = ListLinePlot[EulerMethodPoints[1000, 1/1000, 0, 1, f1]];
```

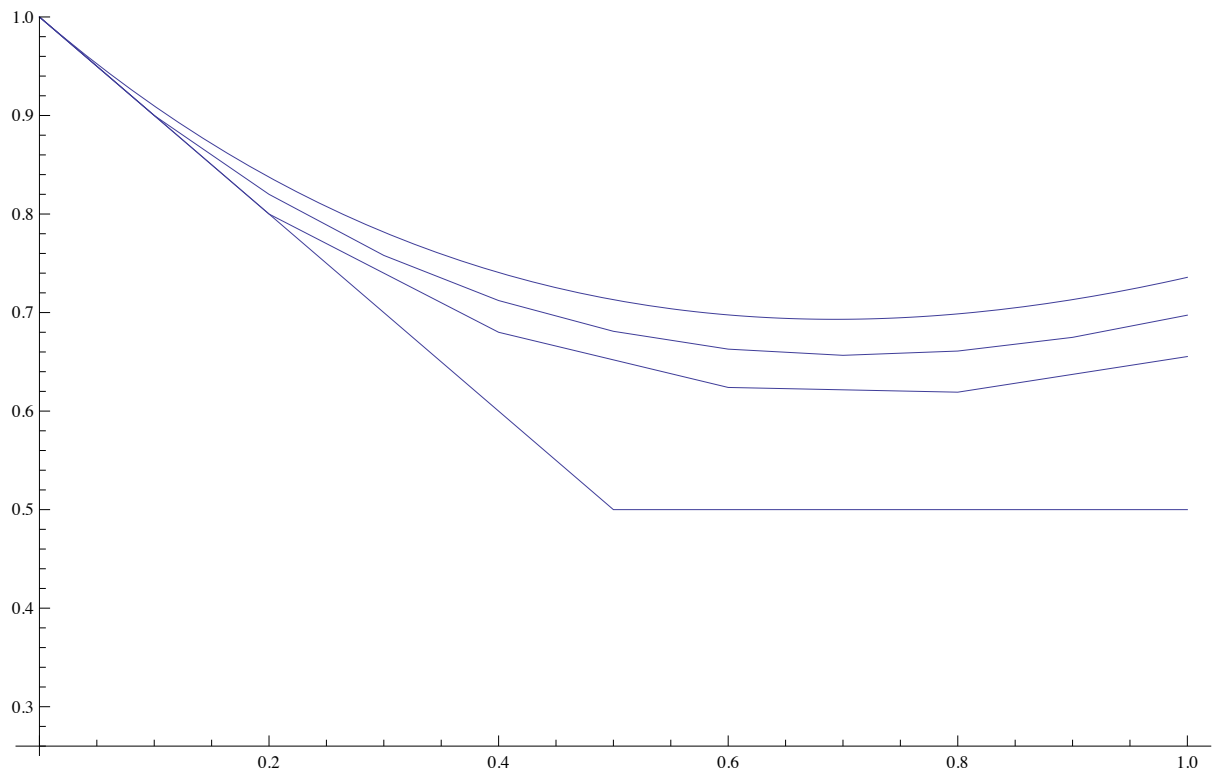
Show[plotexact1, plot1euler2]



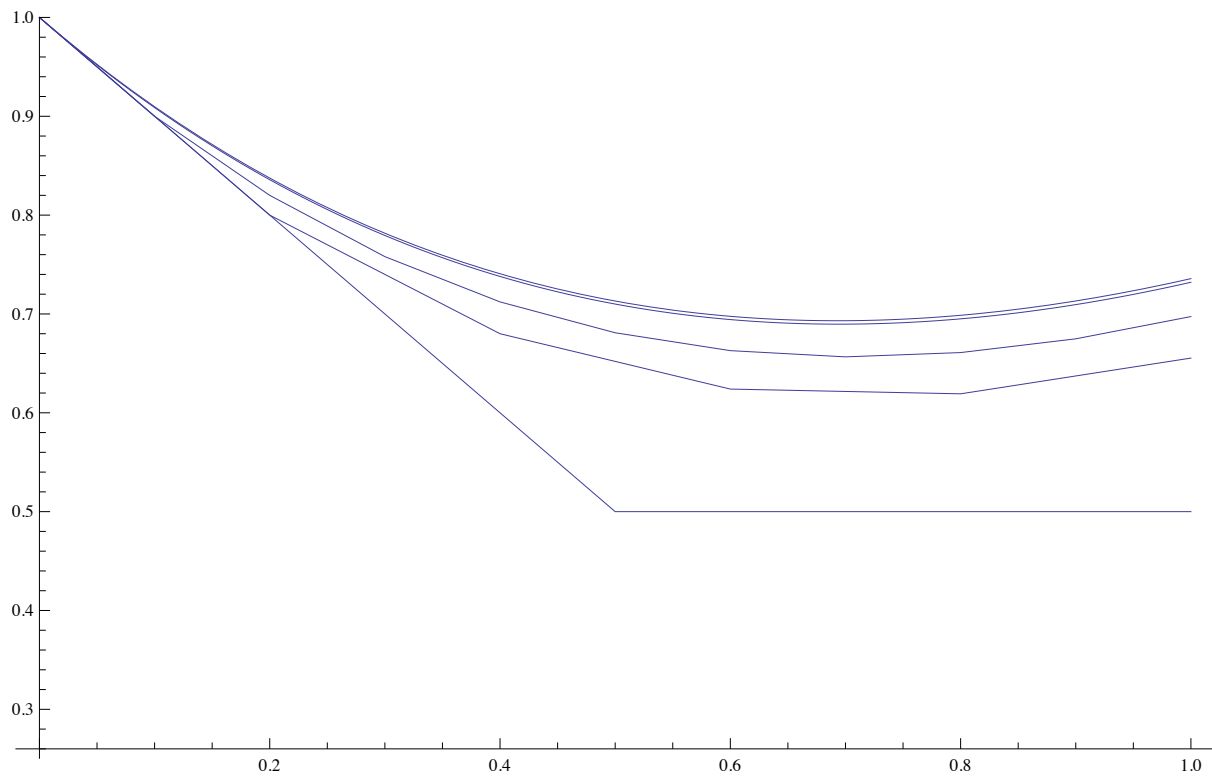
Show[plotexact1, plot1euler2, plot1euler5]



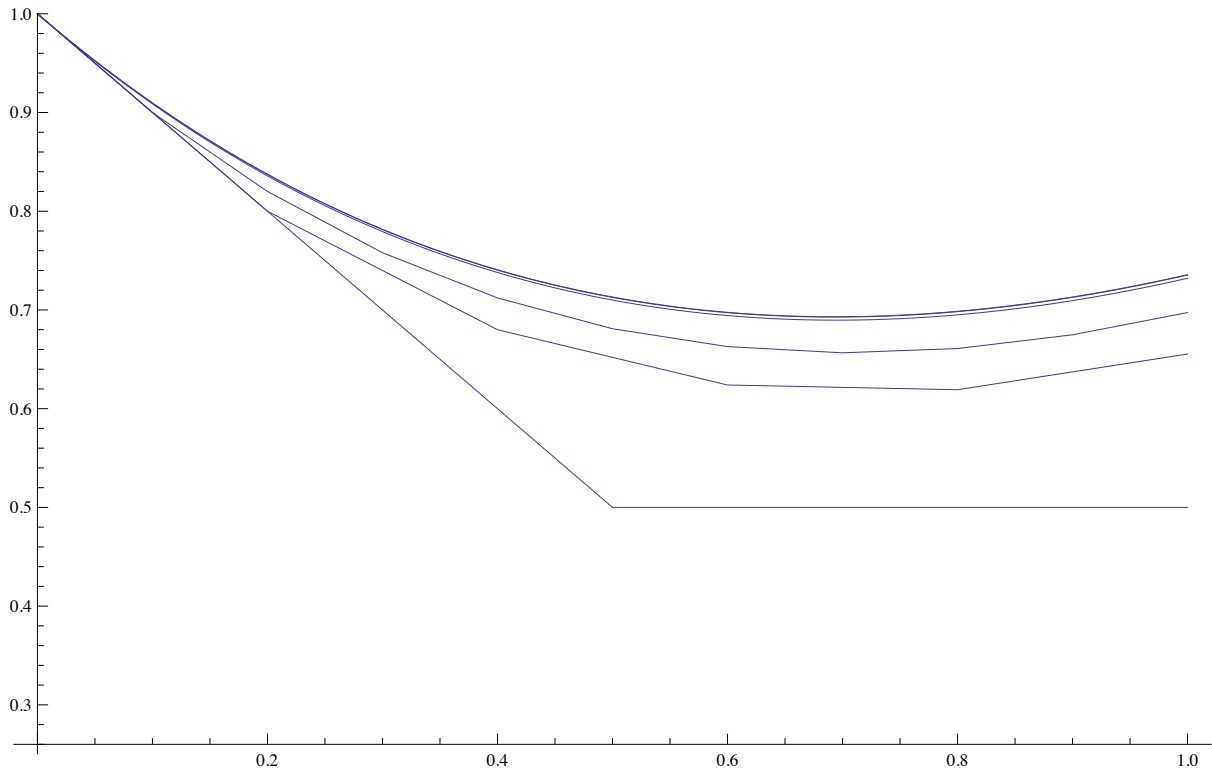
```
Show[plotexact1, plotleuler2, plotleuler5, plotleuler10]
```



```
Show[plotexact1, plotleuler2, plotleuler5, plotleuler10, plotleuler100]
```



```
Show[plotexact1, plotleuler2, plotleuler5,
      plotleuler10, plotleuler100, plotleuler1000]
```



```
finalvalues1 = Map[#, Last[EulerMethodPointsN[#, 1 / #, 0, 1, f1]][[2]]] &,
  {2, 5, 10, 100, 1000, 10 000}
];
TraditionalForm[Grid[Prepend[Map[#[[1]], #[[2]], N[solexact1[1] - #[[2]]]]] &,
  finalvalues1
], {"n", "yn", "error"}, Frame -> All, Alignment -> "."]]
```

n	yn	error
2	0.5	0.235759
5	0.65536	0.0803989
10	0.697357	0.038402
100	0.732065	0.0036942
1000	0.735391	0.000368033
10000	0.735722	0.0000367895

IVP: $y' = \frac{1}{2} y^2$, $y(0) = 1$; estimate $y(2)$

$$f2[x_, y_] = \frac{1}{2} y^2;$$

$$\text{solexact2}[x_] = \frac{2}{2-x};$$

```
plotexact2 = Plot[solexact2[x], {x, 0, 2}, PlotRange -> {{0, 3}, {0, 20}}];
```

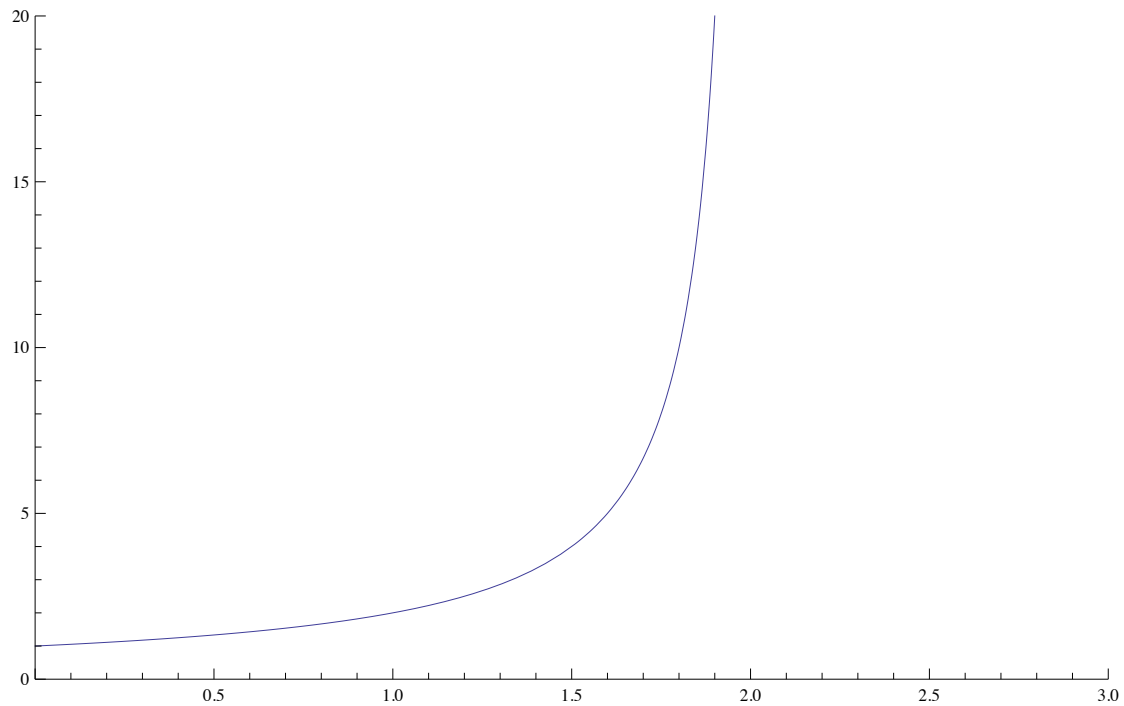
```
plot2euler2 = ListLinePlot[EulerMethodPoints[3, 1, 0, 1, f2]];
```

```
plot2euler4 = ListLinePlot[EulerMethodPoints[6, 1/2, 0, 1, f2]];
```

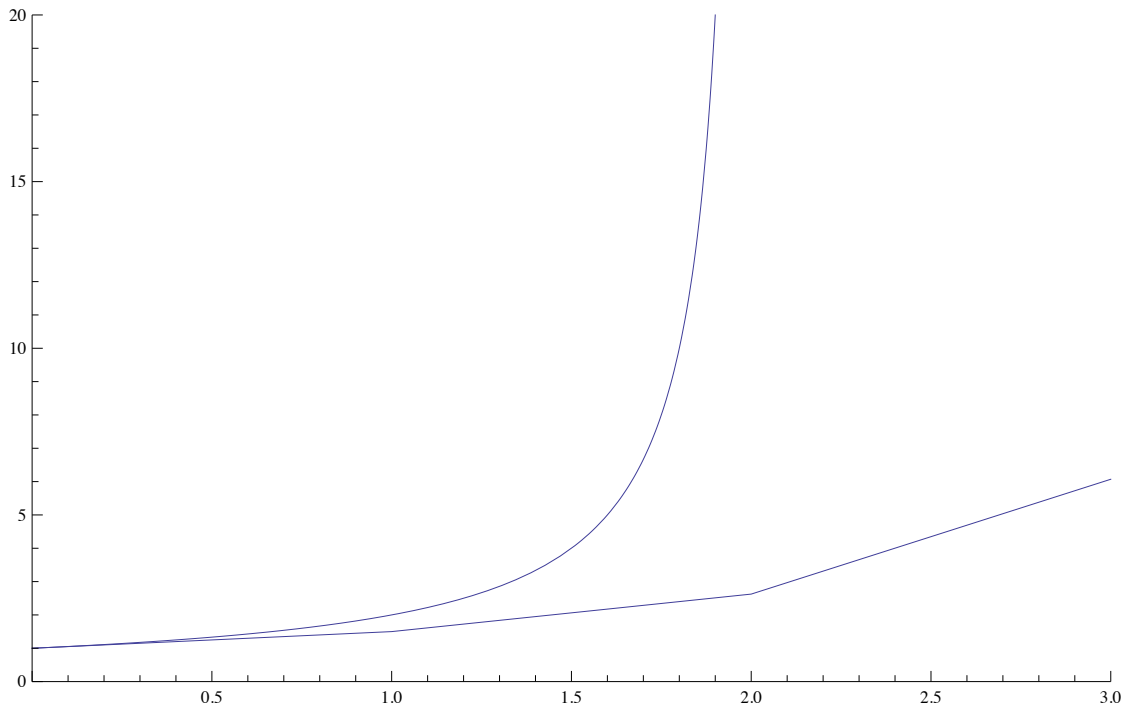
```
plot2euler64 =
```

```
ListLinePlot[EulerMethodPointsN[96, 1/32, 0, 1, f2], PlotRange -> {0, 20}];
```

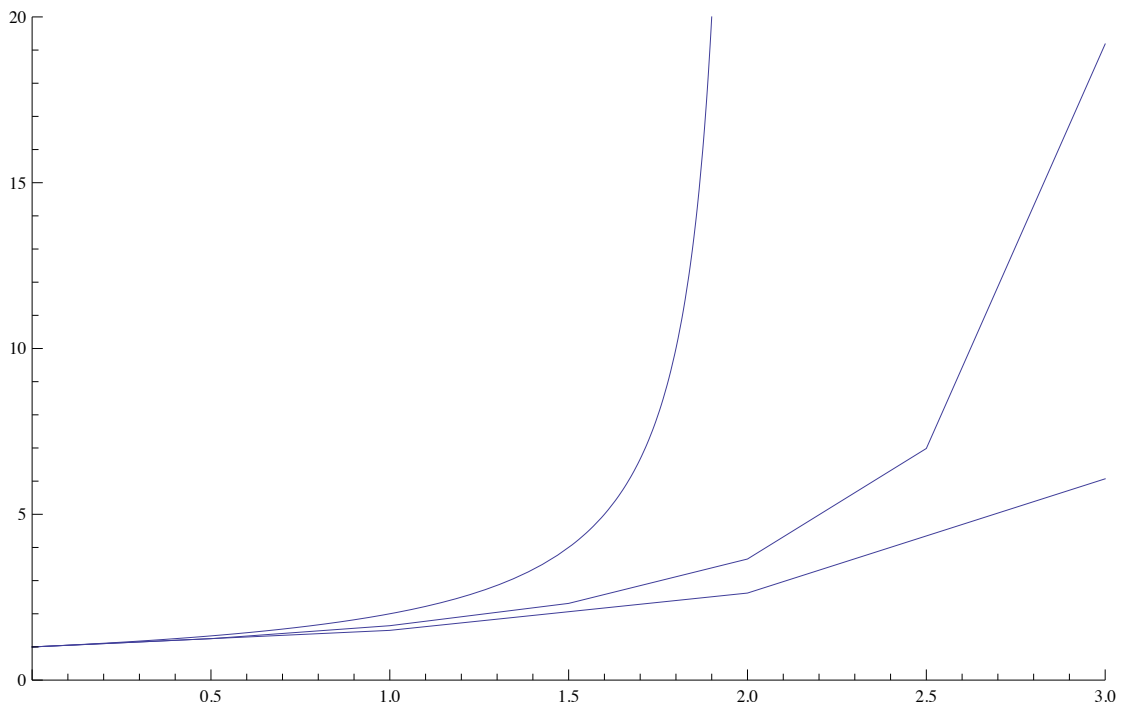
```
Show[plotexact2]
```



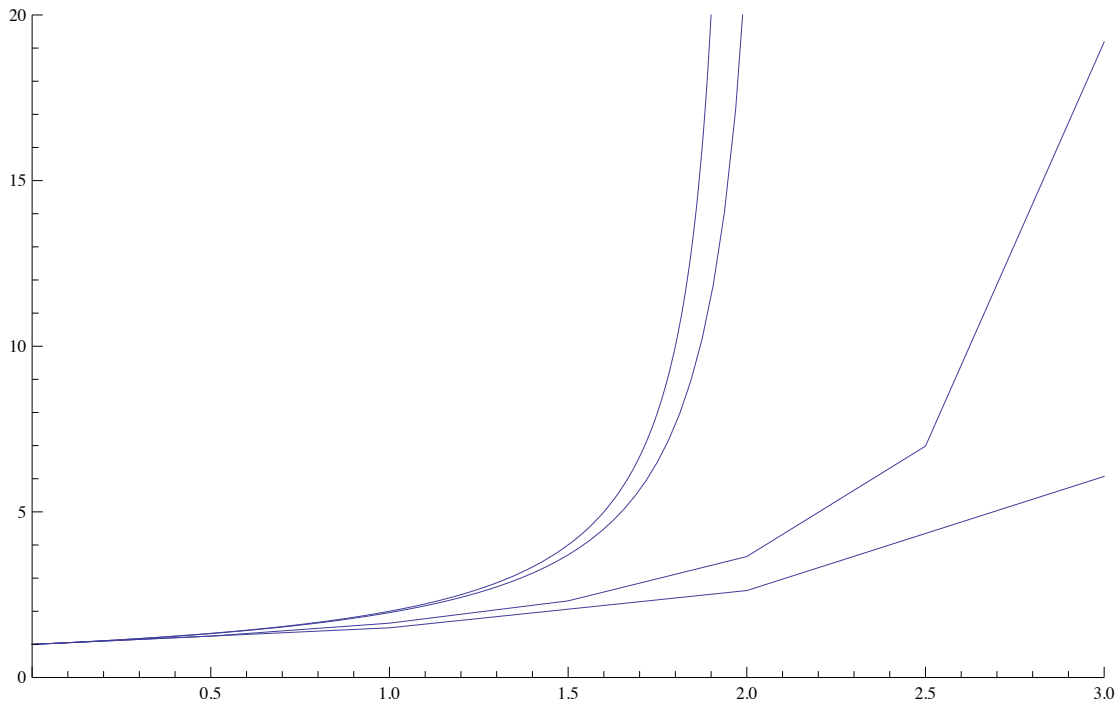
Show[plotexact2, plot2euler2]



Show[plotexact2, plot2euler2, plot2euler4]



```
Show[plotexact2, plot2euler2, plot2euler4, plot2euler64]
```



```
EulerMethodTable[64, 1 / 32, 0, 1, f2]
```

n	xn	yn	yn'
0	0.	1.	0.5
1	0.03125	1.01563	0.515747
2	0.0625	1.03174	0.532246
3	0.09375	1.04837	0.549545
4	0.125	1.06555	0.567696
5	0.15625	1.08329	0.586757
6	0.1875	1.10162	0.606789
7	0.21875	1.12059	0.627857
8	0.25	1.14021	0.650036
9	0.28125	1.16052	0.673405
10	0.3125	1.18156	0.698048
11	0.34375	1.20338	0.72406
12	0.375	1.22601	0.751545
13	0.40625	1.24949	0.780615
14	0.4375	1.27389	0.811393
15	0.46875	1.29924	0.844015
16	0.5	1.32562	0.878631

17	0.53125	1.35307	0.915405
18	0.5625	1.38168	0.954521
19	0.59375	1.41151	0.99618
20	0.625	1.44264	1.04061
21	0.65625	1.47516	1.08805
22	0.6875	1.50916	1.13878
23	0.71875	1.54475	1.19312
24	0.75	1.58203	1.25141
25	0.78125	1.62114	1.31405
26	0.8125	1.6622	1.38146
27	0.84375	1.70537	1.45415
28	0.875	1.75082	1.53268
29	0.90625	1.79871	1.61768
30	0.9375	1.84927	1.70989
31	0.96875	1.9027	1.81013
32	1.	1.95927	1.91936
33	1.03125	2.01925	2.03868
34	1.0625	2.08295	2.16935
35	1.09375	2.15075	2.31286
36	1.125	2.22302	2.47092
37	1.15625	2.30024	2.64555
38	1.1875	2.38291	2.83914
39	1.21875	2.47164	3.05449
40	1.25	2.56709	3.29497
41	1.28125	2.67006	3.5646
42	1.3125	2.78145	3.86823
43	1.34375	2.90233	4.21177
44	1.375	3.03395	4.60243
45	1.40625	3.17778	5.04913
46	1.4375	3.33556	5.56299
47	1.46875	3.50941	6.15796
48	1.5	3.70184	6.85182
49	1.53125	3.91596	7.66737
50	1.5625	4.15557	8.63437
51	1.59375	4.42539	9.79204
52	1.625	4.73139	11.193

53	1.65625	5.08117	12.9092
54	1.6875	5.48458	15.0403
55	1.71875	5.9546	17.7286
56	1.75	6.50861	21.181
57	1.78125	7.17052	25.7082
58	1.8125	7.9739	31.7916
59	1.84375	8.96739	40.207
60	1.875	10.2239	52.2636
61	1.90625	11.8571	70.2954
62	1.9375	14.0538	98.755
63	1.96875	17.1399	146.888
64	2.	21.7302	236.101