Homework #11: Due Wednesday, Apr 24, at 4 PM

Problems are taken from the exercises in the Edwards & Penney textbook. Read through the text before working on the problems, and please make use of office hours provided by the teaching staff or the Math Learning Center if you find them difficult. Submit your homework either to your instructor during lecture or to your TA during recitation or at their office. Late homework assignments will not be accepted.

Problems

Write up these problems neatly and submit them by the due date above. Show your work where appropriate for full credit. Answers without justification may receive no credit, particularly if they are provided in the textbook or student solution guide. If your homework solutions require multiple pages, please staple them together.

- Section 5.2: 10, 12, 24, 28. Omit the graphing on problems 10 and 12.
- Section 5.4: 2, 6, 12. Omit the graphing on problems 2 and 6.
- Additional Problem #1: In problem 4.1.24, we derived the system

  $$m_1 x_1'' = -(k_1 + k_2)x_1 + k_2 x_2, \qquad m_2 x_2'' = k_2 x_1 - (k_2 + k_3)x_2$$

  to model the displacements $x_1(t)$ and $x_2(t)$ of the two masses m_1 and m_2 in the mass-spring system depicted in Figure 4.1.11. Introducing variables $y_1 = x_1'$ and $y_2 = x_2'$ and normalizing, we write this as the first-order system

  $$\begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & 0 & 0 \\ \frac{k_2}{m_2} & -\frac{k_2+k_3}{m_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix}$$

  Assume that $m_1 = m_2 = 1$, $k_1 = k_3 = 2$, and $k_2 = 1$.

  (a) Use the eigenvalue and eigenvector techniques from section 5.2 to find the general solution of this linear system.

  (b) Find the solution matching the initial conditions $x_1(0) = 3$, $x_2(0) = 1$, $x_1'(0) = 0$, and $x_2'(0) = 0$.

  (c) Write the $x_1(t)$ and $x_2(t)$ components of your solution in the form

  $$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \cos(\omega_1 t - \alpha_1)\mathbf{v}_1 + \cos(\omega_2 t - \alpha_2)\mathbf{v}_2$$

  for frequencies ω_i, phases α_i, and amplitude vectors $\mathbf{v}_i$. Interpret each of these two terms with respect to the motion of the masses in the system.