Practice Final Exam: Winter 2007

1. (*40 points*) Find $\frac{dy}{dx}$ for each function. Each answer should be a function of x only.

(a) (*10 points*) $y = \frac{2}{x-1} - \frac{1}{\sqrt{x}}$.

(b) (*10 points*) $y = (\sin x)^{\cos x}$.

(c) (*10 points*) $y = \sqrt{\tan(x^2)}$.

(d) (*10 points*) $y = \frac{(2x+1)^4 \sin(x^2)}{(\ln x)\sqrt{3x-1}}$.

2. (*10 points*) Find the equation of the tangent line to the curve

$$e^{x^2} + e^{y^2} = 2e$$

at the point $(-1, 1)$.

3. (*20 points*) Let

$$f(x) = \ln(x^2 - 1).$$

(a) (*10 points*) You must show all your work, but please write your final answers in the box.

The domain of $f(x)$ is:	__________
$f(x)$ is increasing on:	__________
$f(x)$ is decreasing on:	__________
$f(x)$ has local maxima at:	__________
$f(x)$ has local minima at:	__________
$f(x)$ is concave up on:	__________
$f(x)$ is concave down on:	__________

(b) (*4 points*) Compute the following four limits.

$\lim_{x\to\infty} \ln(x^2-1) =$ $\qquad$ $\lim_{x\to-\infty} \ln(x^2-1) =$

$\lim_{x\to1^+} \ln(x^2-1) =$ $\qquad$ $\lim_{x\to-1^-} \ln(x^2-1) =$

(c) (*1 points*) List all vertical and horizontal asymptotes of $y = \ln(x^2 - 1)$.

(d) (*5 points*) Using your answers from parts (a) and (b), sketch a graph of

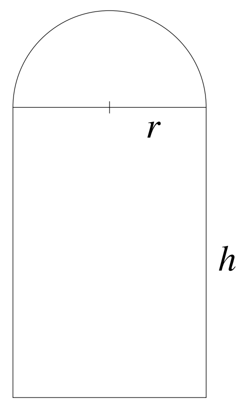
$$f(x) = \ln(x^2 - 1).$$

Even if your answers in parts (a) and (b) are wrong, if your sketch correctly uses those answers, you may earn partial credit.

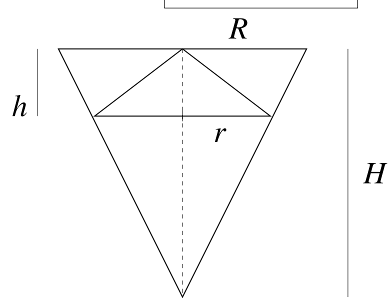
4. (*20 points*) A particle is moving along the curve $x^2 - 4xy - y^2 = -11$. Given that the x-coordinate of the particle is changing at 3 units/second, *how fast is the distance from the particle to the origin changing* when the particle is at the point $(1, 2)$? Hint: As an intermediate step, you should compute the value of $\frac{dy}{dt}$ when $x = 1$ and $y = 2$.

5. (*20 points*) A balloon is rising at a constant speed of 1 m/sec. A girl is cycling along a straight road at a speed of 2 m/sec. When she passes under the balloon it is 3 m above her. How fast is the distance between the girl and the balloon increasing 2 seconds later?

6. (*20 points*) A Norman window consists of a rectangle surmounted by a semicircle, as shown. Given that the total area of the window is $A = 8 + 2\pi$, find the minimum possible perimeter of the window. (Please note the horizontal line between the rectangle and the semicircle does not count as part of the perimeter.) *Hint*: The total area has been carefully chosen so that the minimum perimeter occurs at a very simple value of r. If your optimal value of r is complicated, you have done something incorrectly.



7. (*20 points*) Suppose you have a cone with *constant* height H and *constant* radius R, and you want to put a smaller cone "upside down" inside the larger cone (see figure). If h is the height of the smaller cone, what should h be to maximize the volume of the smaller cone? The optimal value of h will depend on H. Recall that the volume of a cone with base radius r and height h is given by the formula $V = \frac{1}{3}\pi r^2 h$.



8. (*10 points*) For parts (a) and (b), compute the given limits, if they exist. If you assert that a limit does not exist, you need to justify your answer to get full credit.

(a) (*5 points*) $\lim_{x\to\infty} \left(\sqrt{x^2 - 3x + 1} - \sqrt{x^2 + 2}\right)$

(b) (*5 points*) $\lim_{x\to 2} e^{\frac{1}{x-2}}$