Math 19: Calculus **Summer 2010**

Background Practice Problems – Solutions

1. Write as a single fraction:

$$\frac{1}{x+2} + \frac{x}{x^2+5x+6}$$

Solution:

$$\begin{aligned}\frac{1}{x+2} + \frac{x}{x^2+5x+6} &= \frac{1}{x+2} + \frac{x}{(x+2)(x+3)} \\ &= \frac{x+3}{(x+2)(x+3)} + \frac{x}{(x+2)(x+3)} \\ &= \frac{x+3+x}{(x+2)(x+3)} \\ &= \boxed{\frac{2x+3}{(x+2)(x+3)}}\end{aligned}$$

2. Simplify $\frac{\frac{(x+2)^2}{x\cos x}}{\frac{(x+2)\cos x}{x^2}}$.

Solution:

$$\begin{aligned}\frac{\frac{(x+2)^2}{x\cos x}}{\frac{(x+2)\cos x}{x^2}} &= \frac{(x+2)^2}{x\cos x} \cdot \frac{x^2}{(x+2)\cos x} \\ &= \boxed{\frac{x(x+2)}{\cos^2 x}}\end{aligned}$$

3. Simplify $(4^{1/2})(27^{2/3})$.

Solution:

$$\begin{aligned}(4^{1/2})(27^{2/3}) &= (2)((27^{1/3})^2) \\ &= 2(3^2) \\ &= \boxed{18}\end{aligned}$$

4. Find all solutions to $x^2 - 5x = 6$.

Solution:

$$\begin{aligned}x^2 - 5x &= 6 \\ x^2 - 5x - 6 &= 0 \\ (x-6)(x+1) &= 0\end{aligned}$$

so the solutions are $\boxed{x = -1 \text{ and } x = 6.}$ The quadratic formula also produces these values:

$$\begin{aligned} x &= \frac{5 \pm \sqrt{5^2 - 4(1)(-6)}}{2(1)} \\ &= \frac{5 \pm 7}{2} \\ &= \boxed{-1, 6} \end{aligned}$$

5. Find all solutions to $|2 - x| = 5$.

   Solution:

$$\begin{aligned} |2 - x| &= 5 \\ 2 - x &= \pm 5 \\ -x &= 3, -7 \\ x &= \boxed{-3, 7} \end{aligned}$$

6. Find the equation of the line passing through the points $(-1, 0)$ and $(-3, -4)$.

   Solution: The slope is $\frac{0-(-4)}{-1-(-3)} = \frac{4}{2} = 2$.

$$\begin{aligned} y - 0 &= 2(x - (-1)) \\ y &= 2x + 2 \end{aligned}$$

7. Find the equation of the line passing through the point $(1, 1)$ with slope 5.

   Solution:

$$\begin{aligned} y - 1 &= 5(x - 1) \\ y &= 5x - 4 \end{aligned}$$

8. Find the y-intercepts and x-intercepts of $f(x) = x^2 + x - 2$.

   Solution: The y-intercept is $f(0) = -2$. The x-intercepts are the solutions to $x^2 + x - 2 = 0$. Since

$$x^2 + x - 2 = (x + 2)(x - 1),$$

these solutions are $\boxed{x = 1 \text{ and } x = -2.}$ (These solutions can also be found using the quadratic formula.)

9. Find the y-intercepts and x-intercepts of $f(x) = (3 - e^x)(e^x + 1)$.

   Solution: The y-intercept is $f(0) = (3 - e^0)(e^0 + 1) = (3 - 1)(1 + 1) = \boxed{4}$. The x-intercepts are the solutions to $(3 - e^x)(e^x + 1) = 0$. The equation $3 - e^x = 0$ is equivalent to $x = \ln 3$, and $e^x + 1$ is never equal to 0, so the only x-intercept occurs at $\boxed{x = \ln 3.}$

10. If $\cos\theta = 2/3$ for a value of θ between 0 and $\pi/2$, find $\tan\theta$.

    Solution: Since $0 < \theta < \pi/2$, we can draw a right triangle with angle θ. Let the hypotenuse of this triangle be 3. Then the leg adjacent to θ has length 2, and the leg opposite θ has length $\sqrt{3^2 - 2^2} = \sqrt{5}$. Then

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \boxed{\frac{\sqrt{5}}{2}}$$

11. We know that $\sin(\pi/2) = 1$. Find $\sin(\pi/3)$, $\cos(3\pi/4)$, and $\cot(\pi/6)$.

    Solution:

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}, \quad \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}, \quad \cot\left(\frac{\pi}{6}\right) = \frac{\cos\pi/6}{\sin\pi/6} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}.$$

12. Find all solutions to $e^{x^2-1} = 1$.

    Solution: Applying ln to both sides yields $x^2 - 1 = \ln 1 = 0$. Then $x^2 = 1$, so $\boxed{x = 1 \text{ or } x = -1.}$

13. Simplify $\ln(xe^x\sqrt{x+1})$.

    Solution:

$$\begin{aligned}\ln(xe^x\sqrt{x+1}) &= \ln x + \ln e^x + \ln\sqrt{x+1} \\ &= \boxed{\ln x + x + \frac{1}{2}\ln(x+1)}\end{aligned}$$

14. Write the domain of $\ln(x^2 - 4)$ in interval notation.

    Solution: The domain of ln is all positive numbers, so the domain of ln consists of all numbers x so that $x^2 - 4 > 0$. Then $x^2 > 4$, so the domain is $\boxed{(-\infty, -2) \cup (2, +\infty).}$

15. Let $f(x) = x^2$ and $g(x) = 3x + 1$. Compute $(f \circ g)(2)$.

    Solution: $g(2) = 3 \cdot 2 + 1 = 7$, so $\boxed{(f \circ g)(2) = f(7) = 7^2 = 49.}$

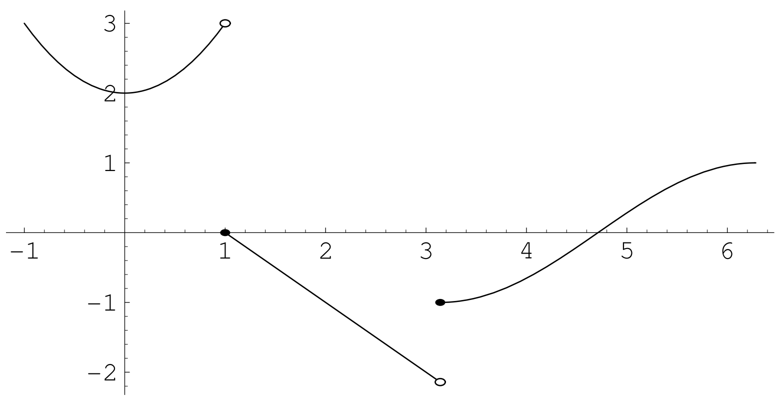
16. Write $\sin(e^{x+1})$ as a composition of elementary functions.

Solution: Let $g(x) = x + 1$. Then

$$\sin(e^{x+1}) = \sin(e^{g(x)}) = (\sin \circ \exp \circ g)(x).$$

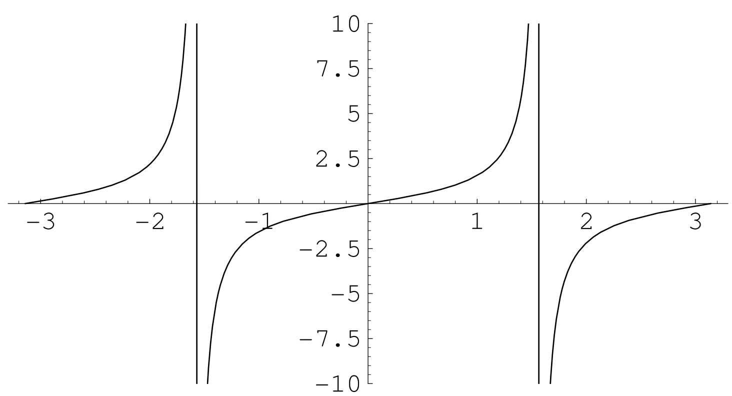
17. Graph $f(x) = \left\{ \begin{matrix} x^2 + 2, & x < 1, \\ -x + 1, & 1 \leq x < \pi, \\ \cos x, & x \geq \pi \end{matrix} \right\}$ on the interval $(-1, 2\pi)$.

Solution:



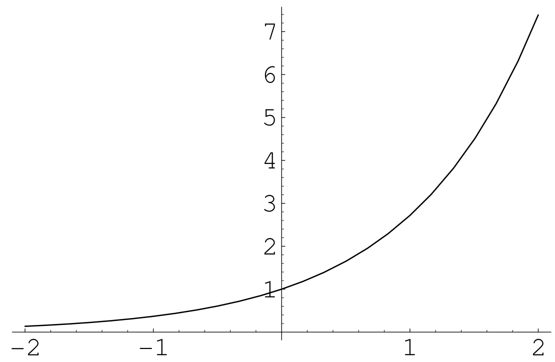
18. Draw the graph of $y = \tan x$ on the interval $(-\pi, \pi)$.

Solution:



19. Draw the graphs of $y = e^x$ and $y = 1/x$.

Solution: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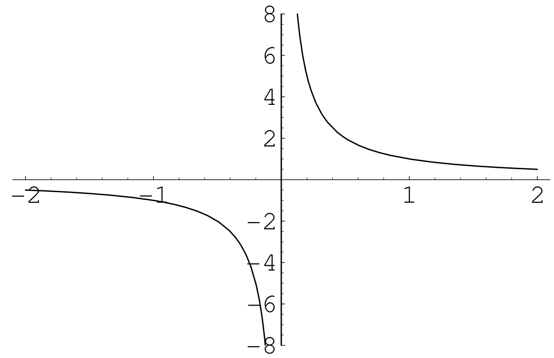
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