Quiz 3 – Solutions

1. (2 points) Let $f(x)$ be a function. Write down a formula which defines its derivative at the number a.

   Solution.

   $$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}.$$

2. *(2 points)* Let $f(x) = 2x$. Use the definition in the previous problem to compute $f'(1)$ (the derivative of $f(x)$ at the number 1). Show your calculations.

   Solution.

   $$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{2(1+h) - 2}{h} = \lim_{h\to 0} \frac{2h}{h} = \lim_{h\to 0} 2 = 2.$$

   or

   $$f'(1) = \lim_{x\to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1} \frac{2x - 2}{x - 1} = \lim_{x\to 1} \frac{2(x-1)}{x - 1} = \lim_{x\to 1} 2 = 2.$$

3. *(2 points)* Use your answer in the previous problem to find the equation of the tangent line to $f(x)$ at the point $(1, 2)$.

   Solution. From previous problem we know the slope of the tangent line at the point $(1, 2)$ is 2. By the point-slope formula, the equation of the line is

   $$y - 2 = 2(x - 1)$$

   which simplifies to

   $$y = 2x.$$

4. (4 points) For the function $f(x) = 2x$, compute $f(x)$ and $f''(x)$.

Solution.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0}(2x + h) = 2x \end{aligned}$$

and

$$f''(x) = \lim_{h\to 0} \frac{f'(x+h) - f'(x)}{h} = \lim_{h\to 0} \frac{2(x+h) - 2x}{h} = \lim_{h\to 0} \frac{2h}{h} = \lim_{h\to 0} 2 = 2.$$