Quiz 6 — Wednesday, August 4

Name: Solution Key

1. (*1 points*) What is $\frac{d}{dx}(\tan^{-1}(x))$?

Solution: $\frac{d}{dx}(\tan^{-1}(x)) = \boxed{\frac{1}{1+x^2}.}$

2. (*3 points*) If $\sqrt{x} + \sqrt{y} = 2$, use implicit differentiation to find $\frac{dy}{dx}$.

Solution: Differentiating implicitly, we obtain the equation

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0.$$

Separating out the $\frac{dy}{dx}$ terms, we have

$$\frac{1}{2\sqrt{y}}\frac{dy}{dx} = -\frac{1}{2\sqrt{x}} \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{2\sqrt{y}}{2\sqrt{x}} = \boxed{-\sqrt{\frac{y}{x}}.}$$

3. (*3 points*) Let $f(x) = x^{(x^2)}$. Compute $f'(x)$ using logarithmic differentiation.

Solution: We take logarithms of both sides to get

$$\ln(f(x)) = \ln\left(x^{(x^2)}\right) = x^2 \ln x.$$

Differentiating with respect to x, we have

$$\frac{f'(x)}{f(x)} = 2x \ln x + x^2 \frac{1}{x} = 2x \ln x + x,$$

so therefore

$$f'(x) = f(x)(2x \ln x + x) = \boxed{x^{(x^2)}(2x \ln x + x).}$$

4. (*3 points*) Suppose that a cube has a side length s that varies with time. At one point in time, the side is 3 cm long and is decreasing at a rate of $\frac{1}{6}$ cm/s. How fast is the surface area of the cube changing? (Make sure to include units in your final answer.)

Solution: Since the box has 6 square sides, each with side length s, the total surface area is

$$A = 6s^2.$$

Differentiating with respect to the time variable t, we have

$$\frac{dA}{dt} = 6(2s)\frac{ds}{dt} = 12s\frac{ds}{dt}.$$

When $s = 3$ cm and $\frac{ds}{dt} = -\frac{1}{6}$ cm/s (since the side length is *de*creasing), then

$$\frac{dA}{dt} = (12)(3 \text{ cm})\left(-\frac{1}{6}\frac{\text{cm}}{\text{s}}\right) = \boxed{-6\,\frac{\text{cm}^2}{\text{s}}.}$$