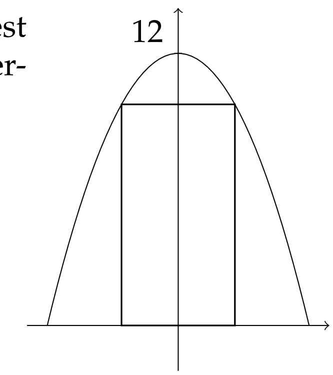
Quiz 7 — Wednesday, August 11

Name: Solution Key

1. (*4 points*) Find the dimensions of the rectangle of largest area that has its base along the x-axis and its other two vertices on the parabola $y = 12 - x^2$ above the x-axis.



Solution: Let x denote the length of half of the base of the rectangle. Then the height of the rectangle is $y = 12 - x^2$, so the area is

$$A = (2x)(y) = (2x)(12 - x^2) = 24x - 2x^3.$$

Differentiating, we have

$$A'(x) = 24 - 6x^2.$$

We look for the critical points of A: setting $A'(x) = 0$, we have $24 = 6x^2$, so $x = \pm 2$. Since both x and y must be nonnegative, the range of valid x-values is $[0, \sqrt{12}]$. Therefore, $x = 2$ is the only critical point that we consider.

Finally, since we are maximizing A on a closed interval of x-values, we check $A(x)$ at $x = 2$ and at the endpoints $x = 0$ and $x = \sqrt{12}$:

$$\begin{aligned} A(0) &= A(\sqrt{12}) = 0 \\ A(2) &= 2(2)(12 - 2^2) = 2(2)(8) = 32. \end{aligned}$$

We conclude that $x = 2$ gives the maximum area, at which point the rectangle is 4 units wide and 8 units high.

2. (*3 points*) Use a linear approximation to estimate $\sqrt{8.94}$. Show your work.

Solution: Let $f(x) = \sqrt{x}$. Since 8.94 is close to the perfect square 9, we let $a = 9$. Then $f(9) = 3$, and since

$$f'(x) = \frac{1}{2\sqrt{x}},$$

$f'(3) = \frac{1}{2(3)} = \frac{1}{6}$. Therefore, the linear approximation is

$$L(x) = 3 + \frac{1}{6}(x - 9).$$

At $x = 8.94$,

$$L(8.94) = 3 + \frac{1}{6}(8.94 - 9) = 3 + \frac{1}{6}(-0.06) = 3 - 0.01 = 2.99.$$

3. (*3 points*) Compute $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\tan 7x}$.

Solution: Trying direct substitution of $x = 0$ gives $\dfrac{\sin 3(0)}{\tan 7(0)} = \dfrac{0}{0}$, so we apply l'Hôpital's rule:

$$\lim_{x \to 0} \frac{\sin 3x}{\tan 7x} \overset{\text{l'H}}{=} \lim_{x \to 0} \frac{3\cos 3x}{7\sec^2 7x} = \frac{3\cos 0}{7\sec^2 0} = \frac{3}{7}.$$