# Practice Midterm Problems

1. Circle "True" or "False." No explanation is needed.

(a)   **True**   **False** $f(x) = |x - 2|$ is one-to-one.

(b)   **True**   **False** $\lim\limits_{x \to 5} \left(\dfrac{2x}{x-5} - \dfrac{10}{x-5} \right) = \lim\limits_{x \to 5} \dfrac{2x}{x-5} - \lim\limits_{x \to 5} \dfrac{10}{x-5}$

(c)   **True**   **False**   A function can have infinitely many horizontal asymptotes.

(d)   **True**   **False**   If f is continuous on $[0, 2]$, then f is differentiable on $[0, 2]$.

(e)   **True**   **False**   The nth derivative of $f(x) = e^{2x}$ is $2^n e^{2x}$.

2. The graph of $f(x)$ is shown. Answer the following questions and explain your reasoning:

(a) What is the domain of f?

(b) What is the range of f?

(c) Is f one-to-one?

(d) Where is f not differentiable?

(e) Sketch the graph of $-f(-x) + 1$ on the coordinate system.
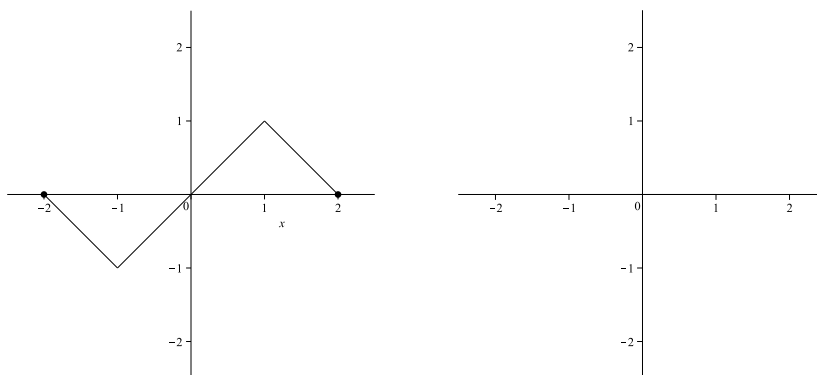
3. For each of the following limits, evaluate it or show it does not exist.

(a) $\lim\limits_{x \to -1} \dfrac{x^2 - 3x - 4}{x + 1}$

(b) $\lim\limits_{x \to \frac{1}{2}} \ln(\sin(\pi x))$

(c) $\lim\limits_{x \to 2} (x^2 - 4)^2 \sin\left(\dfrac{1}{x - 2} \right)$

(d) $\lim\limits_{x \to \infty} \dfrac{3 - x}{x^2 - 3x + 2}$

(e) $\lim\limits_{x \to 0} f(x)$, where $f(x) = \begin{cases} e^x & \text{if } x < 0, \\ 0 & \text{if } x = 0, \\ \tan^2 x + 1 & \text{if } x > 0 \end{cases}$

4. Let $g(t) = \dfrac{t+3}{t-1}$.

    (a) Find the equation(s) of all vertical asymptote(s) of g.

    (b) Find the equation(s) of all horizontal asymptotes of g.

    (c) Find $g^{-1}(t)$.

5. (a) Let $f(x) = x^2 - \sin x$. Compute $f'(x)$.

    (b) Show there exists a number a between $[0, \frac{\pi}{2}]$ such that the graph of $x^2 - \sin x$ has a horizontal tangent line at a.

6. (a) Using the limit definition of the derivative, compute the derivative of $f(x) = 2\sqrt{x}$.

    (b) Find the equation of the tangent line to the curve when $x = 1$.

7. Find the derivatives of the following functions:

    (a) $f(x) = x^5 - x^{3/4} + 1$

    (b) $f(x) = x \ln x$

    (c) $f(x) = \sin(2e^x)$

   (d) $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$

    (e) $f(x) = \ln\left(\dfrac{\sqrt{x}\cot x}{e^x}\right)$

    (f) $f(x) = |x|$

8. The displacement (in centimeters) of a particle moving back and forth along a straight line is given by $s(t) = 2^t + t^3 + 1$, where t is measured in seconds.

    (a) Find the average velocity of the particle from $t = 1$ to $t = 3$.

    (b) Find the instantaneous velocity of the particle at $t = 1$.

    (c) Find the acceleration of the particle at $t = 1$.

9. The figure shows the graphs of f, f', and f''. Identify each curve and explain your choices.
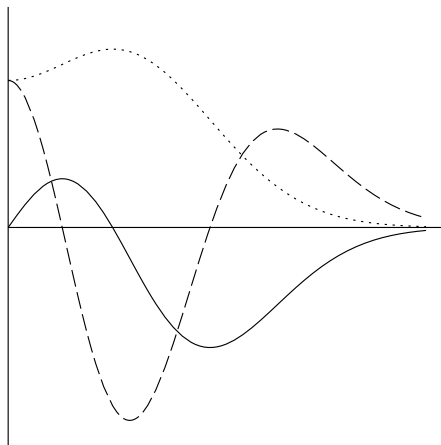
10. Sketch a possible graph of $f(x)$ which satisfies all of the following conditions:

(i) $f(0) = 1$

(ii) $\lim\limits_{x \to -\infty} f(x) = 0$

(iii) $f'(0) = 1$

(iv) f is increasing on $[-1, 1]$

(v) $\lim\limits_{x \to 3^-} f(x) = 5$

(vi) $\lim\limits_{x \to 3^+} f(x) = 2$

(vii) f is decreasing on $[3, \infty)$

(viii) $\lim\limits_{x \to \infty} f(x) = -\infty$